An Analysis of the US Economy from 1950 to 2010

(Text)

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Preface

In the US there is a lot of talk about "the economy", be it by people labeled as "professional economists", be it by "media people" or be it by common citizens. Unfortunately the level of misinformation present in such talk is unbelievably high. It is very sad that some people who label themselves as "economists" appear often to issue grossly inaccurate or misleading statements, which are often accepted as "truths" by the unsuspecting public and by politicians.

In the **first part** of the book (Chapters 1 and 2) we will review what are the factors affecting "the economy'" and some of the *logical* relationships linking these factors. This will lead us to define a specific view of the economy, at a level of detail adequate to properly elucidate the main issues that form the subject of most discussions.

In the **second part** (Chapters (3, 4 and 5) we will use the specified framework to review the historical data of the US economy in the 1950-2010 period.

In the **third** part of the book (Chapters 6, 7 and 8) we will attempt to analyze the above data in order to determine some of the *experimental* relationships linking the primary economic factors.

Of course, such analysis cannot be exhaustive, but we hope to be able to provide some insight on some of the most important issues.

Most of our analysis will cover the 60 years from January 1 1950 to December 31 2009.

An important note on the data on which the analysis is based. The primary source is the Bureau of Economic Analysis of the Department of the Interior. Other important sources are the Department of Labor, the Bureau of the Census, the Department of Energy and the Federal Reserve System. The data for any given year is often published much later than the end of that year. Furthermore, the early published data is often subject to review and reappraisal. At the time of this writing (early 2011), the data for the period 1950-2009 can be assumed to be relatively stable.

Chapter 1

Introduction

What is “the economy”?

We often hear people saying, “the economy is good”, or, alternatively, that “the economy is bad”, but what do they mean by that? Is “goodness” or “badness” of the economy something that we can *measure*, more or less “objectively”? Or is it based just on the personal value judgment of each person?

Clearly the previous questions are not even well defined until we clarify *whose* economy we are talking about. There is no such a thing as “the economy” in the abstract. In order to make sense of the word we must associate it with some *set of people*. At the lower level, each of us is interested in its *personal economy* or, more broadly, in its *household economy*. At the other extreme we may be interested in the *world economy*, although few people’s interests would go beyond their own *national economy*. In the following we will review the major economic environments and some of the methodological issues relayed to economic analysis.

1.1 The Household Economy

Etymologically, the word “economy” means something akin to “household management”, so let’s start at the household level for the time being and ask the question again, what is “the economy” of a household?

But, what is a “household”? In the sense that we will use the word we mean a group of people, normally living together in a single “house”, who function as a single “economic unit”. The members of such a group may differ substantially in the type and amount of contributions to *production* within the household, but they are assumed to enjoy approximately equally the benefits of the *consumption* within the household (although this may be inaccurate, if household servants are included).

Most people would probably answer that the economy of a household includes all those factors that determine the *standard of living* of that household. But this only leads to the next question, namely, what does the “standard of living” consist of? It certainly would include the quantity and quality of our food, the size and quality of our dwelling and things like that. But, would it also include the quality of the air we breathe? Or the density of traffic on the roads we travel? Or the severity of the crime problem in our area? Or the amount of leisure time available to us?

Traditionally the word "economy" has been interpreted in a very narrow sense, to include only the more obvious "material" factors. This might be acceptable if one could claim that such factors are largely independent of other considerations. Clearly this is not the case. If we dedicate a larger fraction of the community gross product to highway construction, therefore improving the traffic conditions, we will automatically decrease the fraction of goods and services available for direct consumption by the households. If we do not include the increased traffic benefit in our definition of the "standard of living", we would have a net loss.

Another aspect of the problem of defining the "standard of living" is the proper accounting of the goods and services that the household produces and consumes within the household itself. Most measures of the material aspects of the household standard of living take into account the goods and services that the household obtains from "the outside". There are a lot of contributions to the "standard of living" of a household that come from activities that are completely contained within the household itself. The values of such contributions are difficult to assess and we will address the issue in more detail later.

The "standard of living' only captures one part of the "economy" of the household, namely the *consumption* of the household, not its *production*. In an industrialized environment, most households "production" consists of "labor" that is sold to "business" or to "government"[1] which in turn compensate the household with wages. The household will use such income to purchase the goods and services it requires. Every household, even the poorest one, owns some *assets*. While such assets might consist only of the clothes on one's back, in many cases they include dwellings and household goods. The typical household may use part of its income to add to its assets. In some cases the household may dispose of some of its assets in order to increase its consumption. The total *net production* of the household will therefore consist of its consumption and its *net addition* (possibly negative) to its assets.

The household derives benefits from the assets it owns. The most obvious case is the one of a household which owns its own dwelling. Such an asset provides the household with shelter that has the same value as what the household would pay in rent for an equivalent dwelling. The same rule applies to other assets that the household might own and use internally for its own benefit, like cars, a vacation home, a boat or other similar assets. Clearly the "equivalent rental/lease values" of such assets are part of both the production and consumption to be allocated to the household. The "invisible income" derived from such assets must be properly accounted for.

[1] It is of some interest to comment on the use of the word "government" in US English. It is commonly used to indicate what in all other Western European (including the United Kingdom!!) is indicated by the word "state" (or its etymologically equivalent terms, "state" in the United Kingdom, "etat" in France, "stadt" in Germany, "estado" in Spain, "stato" in Italy). In Western Europe the term "government" (or its etymologically equivalent terms) is used to indicate what in the US is referred to as the "administration". So in the United Kingdom there is reference to the "the Brown government", in France to the "le gouvernment Sarkozy", in Spain to "el gobierno Zapatero", in Italy to "il governo Berlusconi" (I am not familiar with the corresponding term in Germany). The reason for the difference between "British English" and "US English" may be due to the fact that in the US the word "state" was used to designate the ex-colonies that joined to form the "United *States* of America". For practical reasons we have chosen to follow US usage.

The trouble with some of the items listed above is that they may be difficult to ascertain with any reasonable amount of accuracy. This leads us into an impossible dilemma. If we take into account only those factors that are readily ascertainable, we will have a very incomplete view of the situation. If, on the other hand, we try to take into account all relevant factors, we are forced into making a number of assumptions that, although plausible, are certainly open to question.

There is a more fundamental problem that underlies all of the above issues, namely, even if we agree on all of the factors to be taken into account, how do we *measure* the "standard of living"? Is it a single "scalar" quantity, like "temperature", measured by a *single* number, or is it a more complex "vector" quantity, that can only be described by a *set* a numbers, like "position" in space, that requires three numbers to be fully determined (e.g., latitude, longitude and altitude)?

Assume that a two-person household, say A, has a $100,000 total consumption and another two-person household, say B, has a $200,000 total consumption. Does that mean that the "standard of living" of B is *twice* the "standard of living" of A? In other words, even if we limit ourselves to the more narrow "material" aspects of the "standard of living", is it just *synonymous* with "consumption"? Or is it just *related* to consumption? Is a $40,000 a year house truly twice as *pleasurable* as a $20,000 a year house?

Consider again two households, A and B, such that

- A pays $20,000 a year to lease a house and $2,000 a year to lease a car;
- B pays $2,000 a year to lease a house and $20,000 a year to lease a car;

Do we believe that the two households have an "equivalent" standard of living? If we assume that the two households are operating as completely free agents in a free market, one would have to conclude that each has allocated its own available income in the optimum way according to its own sense of values. Therefore they both have *by definition* the "optimal level of individual satisfaction" achievable with the given amount of money. But, is it the *same level?* Housing and transportation meet different needs. Are conveniences in one area truly interchangeable with conveniences in the other?

If we decide to include in the "standard of living" also the issues that are normally referred to as "quality of life" issues, like pollution, crime, traffic and the like, how do we *measure* their contributions? The obvious way is to ask the household how much other consumption it would be willing to give up, in order to improve those other areas. But would such an hypothetical inquiry truly elicit a valid response?

There is another aspect of the problem of appropriately measuring the "standard of living" of a household. The most typical interpretation of the "standard of living" has to do with the *current* value of the household consumption. But certainly this is a myopic perspective. Most households are concerned not only with their *present* state of affairs, but also with the *future* one, at least over a reasonable time span. How do we measure the future of the "standard of living", or, more meaningfully, the *expectations* about such future? In some respects, the assets of a household do provide some information about the potential future consumption. However, for most households, the expectation of the future "standard of living" relies primarily on the *expectation of future earnings* due to continued employment or to "pensions". How to factor this into the current evaluation of the "standard of living" is not obvious.

1.2 The Island Economy

Let's now consider the case of a completely isolated, but fully industrialized community. Think of it as a large, industrialized island, so remote that no communication and exchange exists between itself and any other community.

In an economically primitive society, a community may be viewed just as the ensemble of a large number of interacting households, occasionally exchanging some goods and services with each other. In an economically more elaborate community the situation is more complicated. The development of agriculture, approximately 10000 years ago, led to a large increase in food productivity. This in turn led to two important changes.

The first one was the development of "specialists" who were not primarily engaged in food production. These people depended for their food on the production of others, but they contributed to the overall economy by providing specialized services. Among such people would be *artisans* who made tools of various kinds, *medicine practitioners*[2] who helped heal the people and *storytellers* to entertain them, among others. The second one was the development of a more or less structured governmental function that became responsible for the coordination of some of the activities of the community (like the common defense of their territory) and presided (among other things) over the collection and redistribution of the agricultural surplus. Jumping all the way to the present industrialized communities, the two developments we mentioned evolved in what we see today.

As we mentioned earlier, today we think of the household primarily as an entity that provides *labor* to *business* and *government*, and receives from business and government goods and services for its own consumption. In other words, the primary entities in the *production* of goods and services are what we have referred to as "business" and "government". Which goods and services are produced by business and which by government varies quite a bit from community to community, even among the so-called western societies that share a considerable historical background. Since our primary interest is the United States of America, we will continue our analysis with the USA structure in mind.

When one attempts to describe "the economy" of a large community it becomes necessary to make a number of decisions about the level of detail to be chosen. What is an appropriate level is of course a function of the type of analysis that one wishes to perform. A low level of detail would provide more complete information, but that might be possibly too much too handle and it may be unnecessary for the purpose of the specific analysis. On the other hand, too coarse a level of detail may hide important issues and may lead to improper conclusions. We will want to select that level that it is most appropriate to issues we would

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[2] Notwithstanding an old cliché, it is almost certain that the first "professionals" were "medicine persons" and not prostitutes.

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like to investigate. However, in getting to that point, it may be appropriate to look at things in a little more detail that strictly necessary, so that we do not accidentally throw away some important factors.

When taking about the "household economy", we introduced the somewhat fuzzy notion of the "standard of living" as the key focus of economic analysis. Does the same concept apply to a community as a whole? It is a perfectly reasonable perspective to claim that in a modern community the roles of "business" and "government" are only to be *intermediaries* in the process of creating the "standard of living" for all the households of the community. In other words, the ultimate objective of the political and economic systems of a community is to produce the best possible "standard of living" for the community as a whole.

But what is the "standard of living" of a community? Assuming that there is a way to measure the standard of living of each household, is the standard of living of a community just the sum of those values? Assume that we have chosen to measure the standard of living of a household as equivalent to their "disposable income" in US dollars. Suppose that a community consists of just two households, A and B, each with an income of $100,000. Now suppose that another community consist of two households, C & D, where C has an income of $50,000 and D has an income of $150,000. The two communities have the same total disposable income. Are the two communities *equivalent* from the point of view of the "community standard of living"?

In more specific terms, is the community standard of living determined solely by the *average* household standard of living, or should we also take into account the differences in the standard of living among different members of the community (i.e., in technical terms, the *variance* of the standard of living)?

We can think of the typical self-contained industrialized economy as structured into three sectors namely, the **household sector**, the **business sector** and the **government sector**. The three sectors interact in a number of ways. However, there is an important distinction between two classes of interactions, namely the distinction between the **real economy** and the **money economy**.

In primitive communities most of the exchanges among households were on the basis of *barter*, i.e. the exchange of some goods and services for other goods and services on an agreed upon basis of "comparative value". Sometimes the exchanges were simultaneous, e.g. a bunch of coconuts for an arrow, while some other times they occurred at different times, i.e. somebody gave something in exchange for a promise to receive something else later. As specialization increased and exchanges became more frequent and involving more diverse sets of goods and services, "money" was invented to simplify such transactions. There are different meanings associated with the word "money". Many people would associate the word "money" with "currency", i.e. with physical tokens with *intrinsic value* or *accepted exchange value*. In our context, we will use the word "money" in its "numeraire" meaning, i.e. as an arbitrary indicator of relative value.

The problem with using money as an indicator of relative value is that it is adequate to compare the value of goods and services *at a given time*, but it may fail to be valid to compare values of goods and services exchanged in transactions occurring at different times. This is the problem of “inflation” and of the need to “adjust” prices to reflect the overall change with time of the value of the “money unit”.

The key point to keep in mind is that “money” is only a convenient intermediary to facilitate the exchange of “real” things, like labor, goods and services. In the end, what is important are the flows of “real” things that determine the “standard of living” of “real” human beings, which is the overall target of economic analysis.

In the **real economy** the household sector provides *labor* to both the business and government sectors. Most of the *goods* and *services* it receives from the business and government sectors are consumed to achieve its desired standard of living. It also consumes some of its own assets, but reinvests some of the received goods into new assets.

The business sector typically produces most of the goods and services. Some of these goods and services flow back to the household sector, some of them flow to the government sector and some of them are re-absorbed by the business sector itself. In such production the business sector consumes some of its assets, but a fraction of the goods produced by the business sector is reinvested by the sector in new assets.

The government sector receives some of the goods and services from the business sector and produces goods and (mostly) services for the household sector. It also consumes some of its assets and reinvests some of its products in its own assets.

The **money economy** is more complex. The business and government sectors pay *wages* to the household sector. In addition, they pay *pensions*, which may be seen as a form of *delayed wages*. The business sector receives *payments* for its goods and services from both the household and government sectors. The business sector also pays dividends to the household sector (which is assumed to be the ultimate owner of all equities). The government receives *taxes* from both the business and household sectors. In addition, the government provides *direct cash transfers* to the household sector in the form of many *income redistribution* programs. In order to maintain a zero overall cash flow, each sector must either provide or accept *cash financing*. This in turn will require that each of the sectors may pay or receive *interests*. Typically the government will be a net *payer* of interests, while the business and household sectors may be either net payer or net receivers, depending on the past history of financial transactions.

In the case of the household economy we mentioned the fact that there are services that are performed within the household and that are not normally taken into account in determining the household standard of living. Similarly, in the case of the economy of a community, there are often goods and services that are *informally* exchanged among households, without being properly “recorded” as formal economic exchanges. We will review this issue a little later in this chapter.

1.3 The National Economy

In the real world there are no communities that can be realistically viewed as completely isolated (with one exception that we will address later). In practice every community interacts with some other communities, which in turns interact with some other communities, creating an interlinked web of interactions that span the whole globe.

However, in most communities the majority of economic transactions are performed within the community itself. This is more so for large communities. Most of our interest is for the economy of the **United States of America** (**USA**). In studying the economy of the USA as a whole it is perfectly reasonable to look at all the other communities with which the USA interacts as a *single community*, that we will label the **Rest-of-the-World** (**ROW**). In other words, from the point of view of economical analysis of the USA economy at a relatively broad level of detail, it is not important:

- to distinguish between transactions between, let's say, the USA and Japan on one hand and the USA and France on the other;
- or to take into account transactions between communities *within* the ROW.

In the remainder of our analysis we will generally consider the world as divided in two, the USA and the ROW.

The economies of both the USA and the ROW can be assumed to be structured in the three sectors we outlined in the previous section, namely the household, business and government sectors. However, in addition to the *domestic* interactions that we described in that section, we will have do deal with a number of *international* interactions between the USA and the ROW.

But, what are the "USA" and the "ROW"? Strangely enough there is considerable confusion in the *definition* of these two entities. Essentially the problem comes from different interpretations of the word "nation". Depending on context, we use the word "nation" to indicate either a *territory* or a *set of people*. In most cases such ambiguity is unimportant or is clarified by the context. However, in economic issues the distinction is very important and, unfortunately, it is not always made clear.

Is the household of a ROW national who happens to reside in the USA a *USA household* or a *ROW household*? Is the business activity of a USA-resident, wholly owned subsidiary of a ROW corporation to be considered a *USA activity* or a *ROW activity*? Is an asset physically located in the USA, but owned by a ROW business or household a *USA asset* or a *ROW asset*?

The choice between the above alternatives (and many other similar ones) should be made so as to meet a number of objectives, some critical, some just desirable. The *critical* objectives to be met are that:

- the choices must be *self-consistent*, so that any element is counted *exactly once*;
- the choices must be *symmetric*, i.e. similar situations between the USA and the ROW should be treated symmetrically.

Some *desirable* objectives are:

- the determination of how an element should be counted should be easy to make;
- the data about the activities involved should be available;

- the *formal* choices should be as much as possible consonant with the *intuitive* choices of the general public.

Unfortunately, some of these objectives may be in conflict with each other.

The way economic data is made available limits our range of choices. For the USA, most of the available data treats all households who are *physically resident* in the USA as "USA households", irrespective of the nationality of the people involved. This is practical for two reasons:

- it would be very difficult to keep track of the nationality of every household in the USA;
- for a household with people of mixed USA and ROW nationalities it would be very difficult to make an assignation.

However, this means that a USA resident household that receives wages from a ROW business enterprise must be perceived as *exporting labor* and *importing wages*. Alternatively, we can assume the existence of a "phantom" USA resident foreign subsidiary that pays the wages and exports services to the ROW, for which it receives payment.

There are a number of different types of international interactions. The most obvious is the *exportation* and *importation* of goods and services between the business sectors of the USA and the ROW. Another form of export/import activity occurs when a USA household purchases a good or service form a ROW business enterprise. If the good or the service is delivered from the ROW business enterprise physically to the USA household in the USA, it may be acknowledged as an international export/import transaction. If however, the household purchases the good or service directly in the ROW territory and consumes it there (e.g. when somebody is in vacation abroad), it will not explicitly appear as an export/import transaction.

Another major problem arises with respect to ownership of real assets. Clearly there are USA households that own assets physically located in ROW territory and viceversa. The first issue has to do with the accuracy with which such information is known. If a ROW resident household moves to the USA as a permanent USA resident household, how are the statistical records going to be updated about the assets that household may still own in the ROW? Similarly, if a USA household purchases in cash an asset physically located in the ROW, in most cases it will not be possible to register such transaction as adding to the ROW resident assets of a USA household. The lack of completeness in the proper handling of international transactions is well known and leads to sizable discrepancies in the official attempts to keep track of such transactions. Even less clear is the accuracy with which we are aware of international transactions that exchange *financial assets*, like bonds and equities. Particularly since some countries explicitly provide a veil of "privacy" over some such transactions.

1.4 The World Economy

Earlier we introduced the notion of the "island economy" as a purely fictitious intermediate step on our way to describe the more realistic "national economy". There are practically no nations that could be truly considered isolated. However, there is one community that can be truly viewed as an "island economy" and that is the **world community** taken as a whole, where by the term "world" we mean the whole planet Earth.

Quite clearly, we on Earth have very few interplanetary transactions with Martians or Venusians. For all practical purposes all transactions can be viewed as transactions between units of the world economy. Similarly, all assets are owned by some world unit.

There is however a major problem in looking at the world economy as a single economic system. In section 1.2 we mentioned the difficulty associated with combining into a single "household sector" households with considerably different income levels. This problem becomes enormously exacerbated in the case of the whole world, in which the differences in economic level of both household and business units can be extreme.

1.5 Economic Measurements

People not familiar with history do not fully appreciate the importance of "measurement" in the development of "scientific methodology". Even before recorded history, we learned how to measure "distance", so we could tell "how long" an object was or "how far" two places were from each other. This led to the development of *geometry*. However, although we have always felt that sometimes it was "cold" and sometimes it was "warm", it was not until the invention of the "thermometer" in the 1700's that we were able to talk about "temperature" in a *numerical* way and therefore be able to say "*how* cold" or "*how* warm" it was. Only after that, we could start to develop *thermodynamics*.

It may be argued that the invention of writing was linked to the need to record the measurements of early *economic variables*. After the agricultural revolution became fully established, many political systems evolved toward a structure in which a "central authority" collected a large part of the agricultural surplus and then redistributed it among the population. To do so, it was important to keep records of what was coming in, what was stored and what was going out. In many cases, early writing was developed to do just that.

Jumping forward about 5000 years, in the 1700's the word "statistik" was introduced in Germany to indicate "a variable of interest to the state". These variables included of course *population variables* (like number of people, their ages and so on), but also included variables that today we would call "*economic variables*", i.e. variables related to the use of resources, the flow of goods and services and the flow of money.

The problem of choosing *what to measure* and *how to measure it* can be a very complex one. Let's consider a simple example. Suppose we believe that it is important to measure *total wheat production* in our community. How do we do that? We may get some data by measuring the quantities of wheat that are offered for sale *by original producers*. We have to be careful not to accidentally count wheat that has already been bought from a producer and is now being resold in the marketplace. However, this is not the whole story. A number of wheat producers may have consumed or stored on their own premises some of their production. Getting a reasonable estimate of that amount may be extremely difficult. In a highly industrialized agriculture, an overwhelming percentage of the total production will be marketed on the open market, making things easier to measure. However, in a less industrialized agriculture, many agricultural producers will consume and store a large part of their own production, making things considerably more difficult.

Another problem is the fact that "wheat" comes in a number of varieties. Is it really acceptable to sum up all the quantities, regardless of their type? If we try to be too precise and we keep each variety's total separate, we are faced with the problem of possibly a higher level of detail that we are really interested in. On the other hand, if the different varieties have considerably different *nutritional value* or *market value*, we may be confusing the issue by summing up all of their totals in a single number. Usually we want to compare the value of a variable in one time frame to the corresponding value in a different time frame. If we sum all of the varieties into a single number and the relative distribution of the varieties remains the same, we are reasonably OK. But what do we do if the relative distribution of the varieties changes with time? Are the two totals really measuring the same thing?

The above example was a relatively easy one, since we can assume that there are not too many grossly different varieties of wheat and that the characteristics of a given variety of wheat will remain essentially constant with time. But what if we want to measure the production of *automobiles*? There are an extremely large number of automobile types. How do we combine them into manageable sets? Also, a given make and model of automobile typically changes from year to year (usually by adding extra features). Is the production count in one year really measuring the same kind of thing as the production count in another year? This problem has become a very sensitive one with "high technology" products, like personal computers, whose technical capabilities have changed extremely fast in recent years. Is it realistic to count one "year-2000-model PC" as the same thing as one "year-1990-model PC"?

The above issues can be partially addressed by selecting to measure each basic variable at a very detailed level, postponing the problem of how to aggregate such variables to a later phase of the analysis. It is always possible to "remove detail" (if unnecessary) by combining variables, but it is not possible to "recreate detail", if the original measurements were made at too coarse a level.

A most difficult problem consists in defining and measuring those variables that are not "in the public record". In all households with children, some of the adults in the household spend part of their time taking care of the children. This is an economically valuable activity. In fact, "rich" people can hire a "nanny" to perform such services, for a fee. It is not easy to evaluate the extent of such services. The only reasonable avenue is through detailed (and costly) interviews with a *selected sample* of "representative" households. The same problem arises for all goods and services that are "bartered" or are provided "free of charge", i.e. without any explicit direct payment, but for which "compensation" may be provided "in kind" at different times.

A similar and more difficult problem is the evaluation of valuable economic activities that are *illegal*. Collecting the relevant information would not only be difficult but outright dangerous in most cases.

A very critical problem is the one of assigning *money values* to many of the *real* economic variables. In a "*market driven*" economy there is often an acceptable definition of *market* *value*, on the basis of the record of “free exchanges” of goods and services. The major problem arises for goods and services that are primarily or exclusively produced and distributed by the *government*. In the latter case the assigning of money values to such goods and services is often based on somewhat arbitrary criteria and open to considerable uncertainty. A similar problem obviously arises for all those goods and services discussed above which are outside the public record.

The issue of *measurement* in the study of the economy is so central to any credible investigation that it is worthwhile to repeat again the key points. There are two major classes of economically significant variables, namely **item quantities** and **item values**.

For *item quantities*:

- first of all we must *select* the variables that we choose to take into consideration; such selection must be *inclusive* of all those variables that we believe to be relevant to the investigation;
- each variable must be *formally defined*, so that its *formal* measurement procedure is *unambiguous*;
- whenever the formal measurement of the variable is impractical, a definite procedure for its *estimate* should be provided;
- each variable should be as much as possible *time invariant*, i.e. its measurements at different times should be acceptable as being fully comparable;

For *item values* the issues are much more complicated. First of all the notion of “value” must be clarified. In many contexts it would be desirable if we could measure the “*use value*” of an item to each specific acquirer of that item. Unfortunately we do not have an even remotely acceptable formal definition for such a thing. The only “value” that we can meaningfully talk about is the **relative exchange value**, i.e. the ratio at which a given item is being exchanged *on average* with other items. We express such value through the item’s **money value**. The units in which such values are expressed are essentially arbitrary, but must be obviously the same for all value items assessed at any one given time. In practice, such values are expressed (in the USA) in terms of “current US$”, since most of the information available for the determination of such values is obviously in those terms. In particular, when the item being measured is a (USA) **domestic money transfer**, its value is obviously measured in “current US$”.

A major problem may arise in the case of **international transactions**. If such transactions are defined also in current USA$, there is no problem. If, however, the transaction is defined in terms of another currency, there is the problem of translating its value in current US$, if that is the unit to be used in the overall analysis.

The major problem with *item values* is the methodology for comparing relative exchange values at some specific time, **t1**, with relative exchange values for the same items at some other time, **t2**. In simpler words, this is the problem of translating values expressed in *current* US$ at different times into values expressed in some kind of *constant* US$. This problem is much thornier than most people presume.

1.6 Economic Theories

Most people do not fully appreciate the difference between *experimental science* and *theoretical science*. The following may be overly “academic”, but I believe it is methodologically important.

The term “experimental” has acquired a broader meaning than its strict etymology would warrant. In its stricter interpretation it would mean “having to do with making experiments”, and observing the results. In its broader meaning it is normally used in conjunction with “systematic observations and measurements”, even when no explicit “controlled experiment” is involved. In economic studies (particularly macroeconomic ones) there is no such a thing as being able to perform a controlled experiment. We must limit ourselves to the careful and systematic observation of the world. Of course, at both the personal, business or governmental levels we can take actions that do affect our economic life at both he individual, national and world levels. However, we cannot “try” different things under “controlled” conditions. There is only one “economic history”. We do not get a second chance at trying something different.

Experimental science fundamentally deals with the observation of the “real world”. *Experimental economics* is what we have been discussing in the previous section. It involves the selection and measurement of “relevant” quantities, with a systematic and rigorous methodology. It is important to note that, broadly speaking, there is no “experimental science” unless there is “measurement”, i.e. the association of *values* with the *descriptions* of observable quantities. Typical *experimental assertions* might go as follows:

The number of people residing in the USA on April 1, 1990 was 248,709,873
The number of people residing in the USA on April 1, 2000 was 281,491,906

Theoretical science is something quite different. The purpose of theoretical science is to create *theories* or *models* of some aspects of the real world. The words “theory” and “model” are essentially interchangeable, although in some cases they have been used with somewhat different connotations. This is not the forum for a detailed discussion of scientific methodology, but it is important to outline the basic issues, albeit in an extremely oversimplified way. The following discussion may appear somewhat pedantic, but there is a reason for it. We will use population data as an example:

- we introduce a *label*, **t**, to indicate the value of “time” and we decide that it can take as values any “**day date**”;

- we then introduce a label for the population of the USA at any specific time, **t**, i.e.

  USA_population(t)

- we will then be able to say that

  USA_population(4/1/1990) = 248,709,873
  USA_population(4/1/2000) = 281,491,906

  which is just a restatement of the experimental assertions mentioned above;

- we may then assert that, based on typical *interpolation* procedures,

USA_population(4/1/1995) = 264,593,682;

this is not a “factual” statement in the same sense that the previous two were. Since we do not have an explicit measurement of the population of the USA on April 1, 1995, the above is a *plausible estimate*, but must be viewed as a *theoretical assertion*, i.e. a different kind of assertion than the previous two; since we cannot go back and *measure* past data, the above assertion is not “confirmable”; however, we probably feel “confident” that if we had performed the measurement, the actual result would not have been very different;

- we may then assert that, based on typical *extrapolation* procedures,

USA_population(4/1/2005) = 299,469,333;

This is obviously also a *theoretical assertion*, but there are two differences with the previous one; first, since it refers to an event in the future we realize that, notwithstanding its plausibility, a number of unusual occurrences might grossly invalidate the *prediction*; second, we have the option of performing the corresponding measurement, at the appropriate date, in order to possibly “confirm” the prediction;

- the most interesting situation occurs, however, when we have measurements about another interesting variable, let’s say

USA_birth_rate(t),

and we go on to define a function **F** such that we choose to assert that**,** for all values of **t2** and **t1**, with **t2** greater **t1**, the following relation obtains

USA_population(t2) = F(t2-t1, USA_population(t1), USA_birth_rate(t1));

this asserts that the population at any given time **t2** can be calculated knowing the population at a previous time, **t1**, and the birth rate at that time. This is an assertion that belongs to quite a different *class* of *theoretical assertions*. We might call them *generalized assertions*. Since they say something about both *past* and *future* values of **t2**, they combine the aspects of both types of assertions mentioned above. Most importantly, they say something about *every possible values* of **t1** and **t2**, which is clearly pragmatically impossible to “confirm” with appropriate measurements.

- it is often said that the role of theoretical science is to “predict” the result of future measurements; this is obviously partly true, but it is only part of the story. There is no value to “prediction” per se. The true value of a generalized assertion, like the one above about the relationship between two different variables, consists in the ***actions*** that might be based on that assertion. If we believe that we might control one variable (the **USA_birth_rate(t1)**, for example), we might than act on that variable in order to affect the value of another variable (the **USA_population(t2)**, for example);

A scientific theory consists of a set of generalized assertions of the type we just outlined. A scientific theory has no direct bearing on "reality". It is a *model of reality*, to be used by people as they see fit. A scientific theory is no more than a "working hypothesis", to be used as long as it appears to be useful and to be discarded when it fails to be so.

The "*degree of validity*" of a scientific theory could be judged on the basis of its degree of consistency with past observations. The "*degree of significance*" of a scientific theory could be judged on the basis of the number of people willing to determine their own course of action of the basis of such a theory. Such willingness is typically based on the "confidence" that people have in the theory's ability to "predict" the results of future observations.

Why have we gone to such lengths in discussing the characteristics of a "*scientific theory*"? People's attitudes toward "scientific theories" range from one extreme to the other. For some people, just labeling something as "science" is sufficient to have them accept it as "absolute truth". For others, labeling something as a "theory" is sufficient to have them disregard it as "just a theory".

"Properly constructed" theories are useful tools. They form the basis for our actions, although often the theory underlying our actions is not made explicit. Unconditional belief in their "truth" is dangerous lunacy.

1.7 The Invisible Economy

Most of the published data about the economy reflects the data for the **visible economy**, i.e. the economic activities that are performed "in the open" and that are therefore properly recorded. As we have already mentioned there are a number of economic activities that are "hidden" for a variety of reasons. They are part of the **invisible economy**. They come in a variety of types, the most important of which we will discuss in the following sections.

1.7.1 The Hidden Household Economy

There are a number of economic activities that the people perform "for themselves" and are never recorded externally. There are a number of important subsets of these activities.

1.7.1.1 Housing Ownership

Many households own their own homes. Assume for simplicity that they own them "outright", i.e. with no outstanding mortgages. They are deriving from such ownership the valuable service of "housing". If they did not own their homes, they would pay to somebody else a "rent".

Consider two situations:

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in the first, household A owns the home in which it lives and so does household B, both homes being of equal value;

in the second, household A owns the home in which household B lives and household B owns the home in which household A lives. Each household pays *rent*, in equal amounts, to the other household;

In the first case there is no exchange of money and the "housing services" are therefore invisible; in the second case there are two exchanges of money, with an identical *net result*, and therefore the "housing services' are recorded as part of the GDP. Clearly this is an accidental anomaly and must be corrected in the economic data.

1.7.1.2 Household Farming

It used to be that "family farms" performed a large fraction of farming activities. Each of these farms produced goods that were "marketed" in the general farming markets, but also produced a lot of agricultural products for their own consumption. Such products were never part of the *visible* economy. In the USA the total amount of such products has clearly diminished significantly over the years and it is now an insignificant part of the GDP. However, the role of such household farming is still very important in the economy of that portion of the rest-of-the-world economy that is still not highly industrialized, as in South Asia, Africa and many areas of South America. In such areas household farming is major element of agricultural production and the corresponding fraction of "invisible agricultural products" is significant.

1.7.1.3 Child Care

Most children are still cared for in the home, particularly in the first years of their life. Many of the adults who are not employed (particularly women) are actually employed in the care of children in their own homes. Such activity is clearly an economically valuable one, since those people who can afford it can hire other people to do it for them, for a wage.

1.7.1.4 Household Chores

In practically all households a certain amount of labor is invested in the normal operation of the household, such as cooking, cleaning, gardening and the like. Again, these are economically valuable activities, for the same reason mentioned above, i.e. that in some case people are paid wages to perform those activities.

1.7.1.5 Do-it-yourself Activities

In many households a certain amount of labor is invested in maintaining and improvement the housing and major durable goods, like automobiles, that may be owned by the household. Again, these are economically valuable activities, for the same reason mentioned above.

1.7.2 The Hidden Cash Economy

There are many small businesses that are able to operate, at least partially, on a "cash basis", in such a way as to "hide" such activities from the tax collector and, consequently, excluding them form the visible economy. We are referring here to activities that would be otherwise perfectly legal. Typical of such activities are activities in direct support of the household such as child care, household care and small home repairs and improvements; exactly those activities that we listed above as often being done by the people directly for themselves.

1.7.3 The Hidden Barter Economy

For thousands of years, people have been exchanging goods and services through *bartering*, i.e. the direct exchange of goods and services without the intermediary of "money". In industrialized societies the amount of bartering that is still being performed is relatively small, but it is still there. A lawyer might provide help to a doctor friend and later may be cared for by its friend, without any money being exchanged. A group of friends may help a neighbor with "barn raising", the favor to be returned in kind at some other time.

1.7.4 The Illegal Economy

There are many activities that are defined by the law as being "illegal", but that from the economical point of view are indistinguishable from similar perfectly legal activities. The services provided by a prostitute are fundamentally similar to the services provided by a hairdresser. The services provided by an illegal drug dealer are essentially the same as those provided by a legal pharmacy. The services provided by an illegal gambling establishment are no different than those provided by a legal one. However, all such services are obviously not recorded and do not appear as part of the visible economy.

1.7.5 The Indirect Economy

In an industrialized society, most people work for a business enterprise, from which they get a wage. Most people also have some form of health insurance. The health insurers provide valuable health services to the people. If the employees pay directly for their insurance, the amount of the premiums appears in the total payments that the household sector makes to the business sector. If the employer pays the premiums, such payments are *internal* to the business sector and do not appear as a component of production or consumption. Obviously, in the second situation the employer has *de-facto* reduced by the appropriate amount the wages it paid to the employees. The *net* flow of money form the business sector from the household sector (*purchase payments* minus *wages*) is the same, but the absolute values of the two terms are different. It is important that the two cases be handled in a fully symmetric way.

1.8 Imputations

The way that some of the above situations are handled is through what are called **imputations**. These consist of "additions" and "corrections" that are made to the "raw data" in order to more fairly reflect the real situation.

In the USA, reasonably accurate estimate are provided for the issues discussed in the "housing ownership" and the "household farming" sections. Estimates are also provided for some of the issues discussed under the heading of the "hidden cash" and "hidden barter" economies, but the accuracy of such estimates may be questionable. A reasonable attempt is also made to estimate the effect of the "indirect economy" mentioned above.

There is *no attempt* to take into account the issues discussed under "child care" or the "household chores"; some account is taken of "do-it-yourself activities". Also, there is *no attempt* to estimate the contributions due to the "illegal economy" (except possibly that some of the latter transactions may appear under "false headings" as part of the legal economy). The non-inclusion of the illegal activities, particularly the illegal international drug traffic, creates a distortion in the *international money transactions* that may be significant.

How significant are these omissions? Since we have only anecdotal evidence for some if the issues, it is not easy to make a sound estimate. For the USA, various guesses have ranged from 10% to 20% of the "official GDP".

1.9 The Problem of Valuation

One of the major issues regarding the correct measurement of **real** economic variables is the problem of assigning a "**value**" to the variable itself. There are two separate issues. The first, to which we have already alluded, is the problem of properly assessing the "value of money" at different times, i.e. the problem of determining the proper "constant dollar" normalization. We will not deal with that problem here. The problem we want to discuss is how to properly assess the "value" of labor, goods and services at any given time, in terms of "current dollars", i.e. in terms of the chosen money unit at the time that the labor, goods and services are exchanged.

1.9.1 The Value of Labor

Let's consider the problem of **labor** first. It would appear that the value of "labor" is, *by definition*, the amount of "wages" that are being paid in exchange for such labor. Unfortunately, things are not so simple. Many business establishments provide for the **health insurance** of their employees. In many cases part or all of the premiums are being paid directly by the employer. Such "benefits" are not accounted for as "wages". They are being paid directly from the "production business establishment" to the "insurance business establishment" and do not appear in the money flow of the households. However, the corresponding "physical health benefits" are actually delivered by the business sector to the household sector, apparently without payment. We would have "proper" accounting if we would do three things, namely

- the wages paid by both the business and government sectors would be *increased* by the amount of health insurance paid by employers in behalf of employees;
- the government sector payments to the business sector would be *reduced* by the amount paid for the health insurance of the government employees;
- the household sector payments to the business sector would be *increased* by the total amount paid for the health insurance by both the government and business sectors;

Obviously the *net* flows for each of the sectors would not change. However, the "value of labor" would increase by the appropriate amount, while the "value of purchases" of business products by the household sector would similarly increase. There are other similar "benefits" that are being paid directly by the employer and do not show in the wages being paid, but the health insurance is by far the most significant, in terms of *concurrent* benefits, i.e. benefits that are being paid for and enjoyed in essentially the same time period.

Many employers also provide for some form of **pension plan** for their employees. In many cases part or all of the costs are being paid directly by the employer and do not appear as "wages" paid to the employee. In this case the benefits deriving from the pension plan are delivered as *direct cash payments* to the retired people and they appear as "transfer payments" by government and business sectors to the household sector. If the economic system were in "steady state", i.e. at a fixed economic level, such payments could be perceived as "effectively delayed wages" and summed up with actual wages. However, the reality is that current pension benefits represent the economic situation of a much earlier period. We would have "proper accounting" if we would do three things, namely

- the wages would be *increased* by the amount that (at least theoretically) the employers (both in business and government) are paying into the "pension funds";
- the same amount would be *accrued* to the household sector's *financial assets* (either equity or money);
- the pension payments to the household sectors would be accounted for as *dividend*, *interests* or *withdrawals* from the newly acquired financial assets.

Traditionally the government sector has offered to its employees better pension plans than the business sector. At the same time, the "officially paid" wages by the government sector have been traditionally somewhat lower than the business sector. The above procedure would demonstrate that the "true wages" of the two sectors are much closer.

A major issue with the proper valuation of labor is the effect due to the payment of taxes. The government activities produce a variety of goods and services that are delivered to the business sector or (mostly) to the household sector. Except for a few fees that are paid directly by the users of such government services, practically all of the "payment" for the government services is done through **taxes**. Taxes are paid by both the business sector and the household sector. However, the amounts paid by each sector are **not proportional** to the **value of the services received** by each sector. The business sector receives a very small percentage of the value of the government activities. The government services provided to the business sector include primarily a portion of the police and fire protection services, the results of some of the research activities sponsored by the government and a portion of the general administrative services. These services could be provided (at least in principle) by the business sector by itself, for itself. By paying for such services the business sector would do the same thing as it does when paying *internally to itself* for those *intermediate goods and services* that *do not count* as part of the business sector production of goods and services. In practice, the business sector tax payments far exceed such amounts. This “extra tax amount” is really a payment for government services that are actually delivered to the household sector. The “proper” way of handling such payments would appear to be to do the following things:

- reduce the business tax payment by the “extra tax amount”;
- increase the wages paid to employees by the same amount;
- increase the payment of household taxes by the same amount.

Unfortunately, this is not enough. This would increase the wages of all business sector employees by some percentage. In order to maintain a proper relationship between the business and government sector *wage rates*, it would be necessary do the following additional things:

- increase the government sector wages by the same *percentage* as the business sector wages;
- increase the household taxes by the same amount as the *total* increase in government wages.

This would leave the overall *net* flows undisturbed, but would provide a “fair” valuation of total labor values.

1.9.2 The Value of Goods and Services

The corrections discussed in the previous section have the side effect that also the total payments from the government and household sectors to the business sector have been “properly” corrected. Therefore the total money payments from those sectors to the business sector can be reasonably assumed (at least in first approximation) to represent the “real value” of the goods and services delivered by the business sector to the other two sectors.

However, we still have the problem of properly assessing the value of the goods and services provided by the government sector.

We need here to address an important issue that has a significant impact on the methodology of valuation. It has to do with the “proper role” that is assigned to the economic sectors. The role of the *business sector* is reasonably unambiguous; it is a “producer” of goods and services for the other two sectors. The role of the household sector is also reasonably clear; it does provide some services to itself, but mainly its role is the one of the “consumer” of the goods and services it receives. The role of the government can be viewed in a couple of different ways. The more “standard” way is to view it as a “giant consumer” that does also produce some goods and services for its own use. In other words, it is an “extension” of the household sector. This is the view underlying the way data about the Gross Domestic Product is structured by the Bureau of Economic Analysis. This view is fundamentally incorrect. The role of the government sector is to be an additional “producer” of goods and services, to complement the role of the business sector. Both the "private" business and the "public" government sectors are "agencies of the people" to provide them with the goods and services that they need. The allocation of the production of certain services to either business or government is a political decision that varies from nation to nation. However, the fundamental ultimate needs of the people are the same. The satisfaction of those needs is what the economic system is about. In our model, we view the government sector as a "second producer" of goods and services, besides the business sector.

The problem with assessing the value of the goods and services provided by the government is that there is no direct relationship between the *delivery* of those goods and services and the *payment* for those services. The government provides most of its services, independently from the actual desires at the people at the individual level. The payment for the government services is by *taxes*, whose amount is not subject to individual choice by the consumer.

One approach to try to assess the *value* of the government goods and services is to assess their *cost*. In order to do that, however, we need to look at how *costs* and *values* are related in the business sector. The reason is that in the private sector we assume that "free market rules" allow us to assert that people payments for goods and services represent, *by definition*, what they are worth.

All *real* production is the result of two main factors, namely *labor* and *assets*. Each of the factors must be "rewarded" for its "services" to the production process. The "rewards" of labor are obvious, namely they are the *properly assessed wages*, as discussed in the previous section. How are *assets* "rewarded" for their "services"? Obviously, what we mean is, how we should reward the *owners* of the assets for allowing their assets to be used? It appears obvious that the owners should expect two things, namely

- to get back their assets in the same condition as they were at the beginning of the production process;
- to get some additional compensation, somehow proportional to the value of their assets.

In other words, the *cost* of production that is associated with the use of the assets consists of two components, namely

- the replacement of the "asset loss" associated with the production (if any);
- the "profit" that must be provided in order to motivate the owners of the assets to permit their use;

The business sector *total profit and loss* balance is a function of two factors, namely:

- the profits and losses associated with the production of *real goods and services* using *real assets;*
- the profits and losses associated with the production *financial services* using *money assets;*

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The latter component includes *interests* (both paid and received) and any *money value changes* due to inflation. These factors must be excluded from the analysis in order to derive the "rewards" for the "real assets".

Let's look at the production of the USA business sector. With regard to the *real assets,* the "asset loss" term is usually reasonably estimated (as will be explicitly discussed in the next chapter). What does the "profit" consist of? There are three components to be taken into account, namely:

- the **net real asset additions** to the production and inventory assets;
- the **net money asset addition**, excluding all interest payments and the effects of inflation;
- the **dividends** paid to the owners of the business sector equities.

We will analyze this in more detail later. For the time being the reader should accept that *historically* the total "profits' paid to the owners of the real assets in the business sector have amounted to approximately 7% of the value of the *production* real assets. It would appear reasonable that in determining the value of the government sector contributions to the GDP one would include a similar rate of return on assets. This is not currently being done. In order to calculate how much of an increase in government GDP that would entail one would have to assess the value of the real assets that the government sector uses for the *production* of its goods and services. This is not trivial, in part because the government sector owns assets that are unique and whose "value" is not easily determined. Using current estimates of the government sector assets, the modification outlined above would yield an increase in the government GDP of an increasing percentage amount over the years, from about 7% in 1950 to about 20% in 2000. The corresponding increase in the total GDP would be obviously less, from less than 1% in 1950 to about 2.5% in 2000. However, this is not the only problem.

In a market economy, under the pressure of competition, it is reasonable to assume that business will use both labor and assets in the most effective way to provide any specific good or service. For the goods and services provided by government there is in general no competition. It is not clear therefore that government will use its labor and assets in the most effective way. This means that the valuation of government goods and services, which is based essentially on their labor *costs,* could be overstated, In other words, if government uses two people to provide a service that business could provide with one (something that we may not be able to ascertain) the value of that service would be set at twice its "true" value. This would suggest that the actual valuation of government services might have to be decreased.

## 1.9.3 The Value of Real Assets

There is very little problem with the *initial* valuation of the household and business assets at the time of their purchase. Their purchase price can be taken as the proper measure of their value, at least when such assets are purchased on the "open market". Some business assets are not purchased, but are manufactured "in house". The valuation of these assets is usually done "at cost", which mat be questionable, but the errors can be considered negligible.
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A problem arises about the estimation of the value of such assets in the years after their purchase. Traditionally, such assets are carried "on the books" at their original purchase price *minus* their *depreciation* (where applicable). In practice, the actual "market value in current dollars" of the assets may actually increase, due to the effects of inflation. In some cases, the value in constant dollars may also increase due to "scarcity" of the particular asset type. The estimation of the "proper" market value may not always be an easy one.

A significant problem exists, however, with the valuation of some government assets. Assets that are purchased on the open market fall in the same situation as the one described above. The difficult problem arises in the valuation of those assets that are "unique" to government, like schools, roads, fighter aircrafts, parks and other similar things. In many cases the estimation of their "proper" value becomes rather arbitrary.

1.10 Conclusions

In this chapter we have addressed some of the issues that create difficulties for the rigorous study of economic issues. Such difficulties are not so overwhelming that a reasonable attempt to a systematic analysis cannot be made. One must bear them in mind, however, when reaching conclusions on the basis of such analysis.

Chapter 2

The Economic System

In this chapter we will analyze the basic structure of the whole **economic system** and the distinction between the **real economy** and the **money economy**.

2.1 What is the "Economic System"

The purpose of the economic system, as indicated in Fig. 2.1, is to transform the *labor* produced by the people, in conjunction with the use of available *assets,* into the *goods and services* that they consume plus *new assets* that may be saved.

If we interpret the term "labor" in the widest possible sense, i.e. to include all human activities, and if we interpret the term "goods and services" also in the widest possible sense, i.e. to include all of the effects of the surrounding environment upon the people, the above would represent just about everything in the world. This may be a little too all-encompassing. On the other hand, if we interpret both terms in the very narrowest sense possible, i.e. to include only those activities, goods and services that are *officially recorded as being exchanged for money*, we may develop a very partial and possibly inconsistent view of the situation. Part of the problem of analysis of the economic system consists in the "appropriate" choice of what to include and what not to include.

In the modeling of the economic system, as in all other modeling endeavors, a key factor is the level of detail which we choose, given the issues that we want to study. If the level of detail is not high enough, we may miss some major aspects of the problem. If the level of detail is too high, we may get lost in a set of lateral issues, not germane to the main points. In general, it is appropriate to select a level of detail that is just a little finer that strictly necessary for the purpose at hand. In this way it is hoped that significant issues will not be overlooked, without creating too much additional complexity.

2.2 Problems in Economic Analysis

One of the problems with the analysis of the economic system is the lack of good data. While there are a large number of "statistics" about economic events, the **reliability** and **consistency** of such data are often questionable. An interesting example comes from international trade figures. Each country reports its "balance of trade" figures, including its *exports* and *imports*. If we add all of the exports and all of the imports, we do not come up to the same total. In particular, the *total of all exports* always comes up to be *lower* than the *total of all imports*, which would mean that the world as a whole has a “trade deficit”. It would appear that the world as a whole must be importing goods from Mars or from the great beyond. Some of these discrepancies may be due to real difficulties in “proper” exchange rate adjustments. However, the fact that they are systematically in the same direction generates some suspicion. It is not unlikely that part of the reason is that most governments, which are responsible for the collection of most data, may have a bias toward making each country look “weak” on the trade front. This can be used to justify protectionist measures that are often demanded by various interests in each country. We will return later to the issue of international trade. The point here is that unfortunately a lot of economic “data” is not reliable.

We all have heard the joke about the man, Tom, who, in the middle of the night, runs into another man, Jack, who is desperately searching for something under a streetlight. “What happened?”, asks Tom. “I lost my wallet.”, says Jack. “Here?”, asks Tom. “No”, says Jack, “over there”, indicating a dark corner on the other side of the street. “Then, why are you searching over here?”, asks Tom. “Because here there is more light”, says Jack.

Jack must have been an economist.

Certain aspects of the economic system do not lend themselves to easy measurements. Unfortunately, this has led many economists to ignore such aspects, even although they may be reasonably presumed to have critical relationships to other observable aspects. In other words they look "where there is more light", rather than where the facts are. A typical example is provided by the economic activities which occur outside the "normal" economic channels, sometimes because no formal transactions are involved, sometimes because the activities are illegal, sometimes because an attempt is made to keep the activities outside the view of the tax collector (although the activities themselves may be legal). If the relationship between these activities and the "official" ones were to be an invariant one, one could assume some fixed ratio and leave it at that. Unfortunately, the relationship is affected by economic conditions and therefore ignoring them can lead to serious misestimates of key aspects of the overall economic system. In other words, there are often problems with the **completeness** of available data.

Problems also exist with the **normalization** of the data. By "normalization" we mean the expression of different measurements, relative to different times or different environments, in terms that allow the measurements to be truly comparable. One problem is the accounting for changes in the value of money (inflation or deflation). A completely different issue has to do with the value of a “unit” of the measured commodity. For some commodities it is possible to compare the unit price at different times (e.g. a specific variety of wheat), since the “intrinsic value” of a ton of wheat can be reasonably assumed to be constant. It is not so obvious what to do for more complex things. A modern car may be more expensive than an older one (in its time), even after correcting for inflation. However, is it the "same" car? Or is it intrinsically a "better" car? Is the additional increase in the “real price” of the car a measure of the fact that “inflation for cars” is higher than the average, or of the fact that the average car in past times did not have, as built-in standard features, the same kind of additional (and costlier) features of a modern car, for example in the area of safety?

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This problem has been particularly obvious in the area of computers. The prices of computer units in real money have been decreasing, while at the same time the "performance" of each unit has been increasing. This has created considerable controversy in the issue of the "proper" determination of the changes in the purchasing power of money.

2.3 Historical Data

Our analysis will be based on the economic data for the USA for the period 1950-2010. More precisely, it will be based on the **quarterly data** from **January 1, 1950 to December 31, 2009.** In the following chapters we will use this data to analyze the history of a number of key economic variables. Too often the emphasis in economic analysis is on the future, often forgetting to check that predicted future relationships are consistent with the observed past. After analyzing the past, we will then be able to more systematically investigate the potential future of the economic system.

Most of the data was derived from data provided by the **Bureau of Economic Analysis** and the **Census Bureau**, both of the Department of Commerce, the **Bureau of Labor Statistics** of the Department of Labor and by other governmental agencies. A reasonable effort was made to check the internal consistency of the data. Some minor discrepancies were resolved in various ways, none prejudicial to the main intent of the analysis.

We will use the **US dollar** as the unit of money for all transactions, domestic or international, from the year 1950 to the year 2010. In few cases we will use **"current dollars"**, i.e. the actual amounts used in the transactions at the time they occurred. In almost all cases we will use "**constant 2010 US dollars**", i.e. the values corrected for the purchasing power of money, using the value of the US dollar as of January 1, 2010 as the unit of reference. The methodology for the determination of the latter values is discussed in detail in Appendix A. We have chosen to use the year 2010 as our base so that all the numbers will appear to be "closer" to our current intuitive sense of the "value of the dollar".

2.4 Microeconomics and Macroeconomics

All economic activities result from decisions taken by *individuals.* The study of such individual activities is usually referred to as *microeconomics*. Unfortunately, there is very little detailed data about such individual activities. Accurate data could only be obtained if a very large sample of individual households were studied in detail over a very long time period. Such study has not been practical. Most of the data that we have is *aggregate* data, i.e. data about the aggregate results of the individual economic activities. The study of such aggregate data is usually referred to as *macroeconomics.* In some cases data is available about the economic activities of *subsets* of economic agents, providing some insights in the behavior of such smaller groups. The analysis in our study will be based on macroeconomic data, at a level appropriate to the issues under discussion.

2.5 The Economic Sectors

For the purpose of our analysis we will consider the overall **USA economic system** as organized in three sectors, namely,

- the **USA household sector**
- the **USA business sector**
- the **USA government sector.**

The economy of the USA cannot be considered in the complete absence of the economy of all other nations with which the USA interacts. We will consider all of the other nations of the world as acting as a *single entity*, which we will refer to as the **Rest-Of-The-World** (**ROW**), and will be represented by

- the **ROW sector**.

It may be important to note that, from the perspective of the USA economy, it only makes sense to look at the rest of the world as a single entity. This is not only because of simplicity, but also because it is the only valid economic analysis. The trade between any two individual nations is of little significance, since in almost all cases, no two nations can provide each other with access to all of the goods and services that each requires. For example, Japan must import oil from the oil producing countries, e.g. Saudi Arabia. Not all of the goods and services that Saudi Arabia needs can be provided by Japan. For example they may choose to import airplanes from the US. To balance the books, Japan must export to the US some goods (e.g. automobiles). Therefore, for this particular set of trades, Japan would have a trade surplus with the US and a trade deficit with Saudi Arabia; Saudi Arabia would have a trade surplus with Japan and a trade deficit with the US; the US would have a trade surplus with Saudi Arabia and a trade deficit with Japan. But, overall, each nation could be in perfect trade balance.

There are two distinct aspects of the economic system that we will refer to as the **real economy** and the **money economy**, which we will discuss later in detail. In order to fully represent the activities of the money economy, we will need to introduce an additional entity, namely,

- the **financial sector**.

We will briefly outline below the major characteristics of each sector.

2.5.1 The USA Household Sector

The **household sector** represents the people of the nation as **producers of labor**, as **ultimate consumers** of all goods and services and as **ultimate owners** of all **private assets**. A "household" is assumed to consist of a set of people that operate essentially as a single "economic unit", from the point of view of *consumption* and *asset ownership*. Within a household it is likely that different members will play significantly different roles as *producers of labor*. This is particularly obvious in a household that includes children. However, it will be assumed that the "standard of living" is the same for each member of

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the household. Typically the people belonging to the same household will live together in a single house. However, a young person living away from the family while going to college would still be considered as belonging to the household of the family that economically supports its studies.

In the USA the pattern of household composition has changed considerably in the 60 years from 1950 to 2010. This makes it difficult to analyze *average* patterns of household economic activity. However this is not a problem as long as we are considering *all households as a total aggregate entity* (i.e. the "*household sector*"), as we are going to do in this and the immediately following chapters. In later chapters we will analyze some issued on the basis of the internal composition of the household sector.

### 2.5.2 The USA Business Sector

The **business sector** represents all of the ***organized private*** **economic activities** of the nation. This includes all agriculture, mining, manufacturing, commerce and other service activities engaged in the *private* production of goods and services. It should be noted that it includes also the activities of the people often referred as "household workers". In other words, all household workers are considered as "private contractors", selling their *services* to the household they work for, utilizing their own *labor* to provide those services.

The business sector produces most of the goods and services that are then consumed by the household sector or by the government sector. However, it also produces goods and services that are then used internally by the business sector itself. Such *intermediate* products are not counted as part of the production of the business sector.

Business units are assumed to physically operate either totally in the USA and be part of the USA Business sector or totally in the ROW. In our model "multinationals" with USA presence are handled as consisting of two separate business units, one in the USA and one in the ROW.

### 2.5.3 The USA Government Sector

The **government sector** represents all of the ***organized public*** **economic activities** performed by all levels of government, i.e. the **federal government**, the **state governments** and the **local governments.** In addition we include under that label also all ***unstructured*** **"public" resources and activities** (i.e. things like the "environment"). The government assets will include all lands, buildings, roads, schools and other natural or man made assets that are directly utilized by the government to provide its services. In addition, we will assume that they include all of those *natural resources* that are "held in common" by the people, like air and waters. We will distinguish between the **defense activities** and the **civilian activities**, which play different roles in the economy.

### 2.5.4 The ROW Sector

If we were interested in analyzing the internal operation of the ROW at the same level of detail that we will use in analyzing the USA activities, we should organize also the ROW in a "ROW household sector", a "ROW business sector" and a "ROW government sector".
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However, we are only interested in the external interactions of the ROW with the USA, therefore we will summarize in the **ROW sector** *all* of its economic activities in a single bundle.

2.5.5 The Financial Sector

The purpose of the **financial sector** is to act as a “clearing house” for all the money and equity exchange activities that are part of the *money economy*. We will be able to explain its role more fully in a later section.

2.6 Real and Financial Assets

One of the key distinctions that must be made is between **real assets** and **financial assets**. By **real assets** we mean primarily “hard assets”, like

- **buildings**
- **cultivated fields**
- **mines**
- **manufacturing facilities**
- **office equipment**

and the like. However, we mean also to include those “soft assets” that have a clear marketable value, like

- **patents**
- **computer programs**
- **product designs**

and similar things, to the extent that these assets are not already “included” in some “hard asset”.

The total of the “real assets” in the world represents the “**total real wealth**” of the world.

We distinguish two types of **financial assets**, namely **money assets** and **equity assets**. By **money assets** we mean

- **physical currency notes and coins**
- **traveler checks**
- **checks**
- **checking and savings accounts**
- **credit card balances**
- **loans**
- **bonds**
- **commercial paper**

and other similar objects. Such assets have no *intrinsic value* (with the possible exception of some types of coins). They **represent money units** that are “owed” by somebody to somebody else. Money assets are represented in the form of **money instruments** and they always appear in *pairs*, i.e. a *positive instrument* that is held by someone and an equal valued *negative instrument* that is held by someone else. **The sum total of all money assets in the world is always *exactly zero*.** In order to make the above strictly correct, we will assume that all currency notes and coins issued by governments are to be viewed as representing a "debt" that those governments owe to the owner of the currency note or coin. This is not strictly so from a legal point of view, but it is a convenient assumption. Since the total of the money assets in the world must be exactly zero, the world as a whole cannot "either a creditor or a debtor be".

For each money instrument there is a de-facto "guarantor entity" (often a government) that is responsible for the "validity" of the instrument and the "fulfillment" of the debt. Money assets are expressed in units of some **national money unit**, which in the case of the USA is the **US dollar** (**US$**). We will distinguish between **USA money assets** and **ROW money assets**, but we will actually assume that all such assets are denominated in **US dollars**.

By **equity assets** we mean primarily

- **corporation shares**
- **real estate deeds**

and other similar objects. These assets also have no intrinsic value (they are just pieces of paper). They **represent the (total or partial) ownership of real assets**, which may be physically held for use by somebody else. Equity assets are represented by **equity instruments** that also are assumed to appear in *pairs*, i.e. a *positive instrument* that is held by the *ultimate owner* of the real asset in question and a *negative instrument* that is held by the entity physically holding the real asset. **The sum total of all equity assets in the world is always *exactly zero*.**

We will assume that business enterprises (both in the USA and in the ROW) are fully owned by households or by government agencies. This means that the business sector, *by definition*, will have equity assets equal in value to the negative value of the sum of the value of their *real assets* plus the net total value of their *money assets*. The corresponding positive equity assets are owned by either the USA Household sector, the USA Government sector or are part of the ROW Total Assets. We will consider as **USA Equity Assets** those assets that refer to business enterprises considered part of the USA Business sector or real estate physically located in the USA. We will consider as **ROW Equity Assets** those assets that refer to business enterprises considered part of the ROW Sector or to real estate physically located in the ROW.

2.7 The Real Economy

The casual reader might want to skip portions of this section on the *real economy* and of the following one on the *money economy* and later refer to them, as might be needed for clarification.

The purpose of the economic system is to transform people's *labor* and *assets* into the *goods and services that are consumed* and into *net asset additions* (possibly negative). This is the role of **real economy**. The flows of activities are depicted in Fig. 2.2. The variables are labeled in a consistent way so that the labels have mnemonic value.

In any modern economy, and certainly in any modern *highly industrialized* economy, there are two key factors that lead to the production of goods and services, namely,

- **labor**
- **real assets**,

The **US Resident people** provide their **US Total Production Labor** (**UTPL**) to the household sector. We assume that *every working-age person* in the US contribute its own *total* labor to the household sector. The household sector uses some of that labor itself as **US Household Labor to US Household (UHLUH)**. This is the labor of all working-age people that are **not employed**, plus any additional labor that *employed* people may be providing directly to the household. The remainder of the labor (i.e. that *formally* provided by all *employed* people) is provided by the household sector to the business sector as **US Household Labor to US Business (UHLUI)** and to the government sector as **US Household Labor to US Government (UHLUG)**. We will "measure" the labor by just talking into consideration the number of people employed, assuming that each such person works a "standard" number of hours per year.

The labor contributed to the government sector comes under two headings, namely the **defense labor** and the **civilian labor**. There is a potential source of terminological confusion here, since the "defense employment" is often itself subdivided into the "military employment" (including all of the Armed Forces personnel) and the "defense civilian employment" (which includes all civilians in a defense support role). In our terminology, the "civilian employment" includes only those people employed in *non-defense* related activities (to the extent that such distinction is reflected in available data).

The **business sector** has available to itself its **US Business Production Assets** (**UIPA**). Such assets include all agricultural lands, mines, buildings, manufacturing equipment and in general any valuable production hardware and software. In addition it also keeps some of its previous production as **US Business Inventory Assets** (**UIIA**). The business sector uses its production assets, together with the labor it receives, to produce its **US Business Total Production** (**UITP**). Some of that production is used to add to its own assets the **US Business Products to Production Assets** (**UIPPA**). However, in the course of production, some of its assets are used up, namely the **US Business Production Assets Loss** (**UIPAL**). The US business sector also receives from the ROW sector some products, namely the **ROW Products to US** (**WPU**) and from the US Government sector the **US Government Civilian Products to US Business** (**UGCPUI**). The business sector delivers products to the household sector, the **US Business Product to US Household** (**UIPUH**), to the government sector, the **US Business Products to US Government** (**UIPUG**) and to the ROW sector, the **US Products to ROW** (**UPW**). The balance of the production (possibly negative) is added to its inventory as **US Business Product to Inventory Assets** (**UIPIA**).

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The **government sector** has available to itself two sets of assets, namely the **US Government Civilian Assets** (**UGCA**) and the **US Government Defense Assets** (**UGDA**). The government sector uses these assets, together with the labor it receives, to produce the **US Government Total Production** (**UGTP**). It also receives products from the business sector, as outlined above. Some of the total products are reinvested in its assets, namely the **US Government Products to Civilian Assets** (**UGPCA**) and the **US Government Products to Defense Assets** (**UGPDA**). Some products are delivered to the US Business sector, as outlined above. The remainder is divided between the **US Government Civilian Products to US Household** (**UGCPUH**), the **US Government Defense Services to US People** (**UGDSUP**) and the products to US Business mentioned above. In the course of the government activities, some assets are used up, namely the **US Government Civilian Assets Loss** (**UGCAL**) and the **US Government Defense Assets Loss** (**UGDAL**). Note that we are assuming that all the "benefits" deriving from the defense activities of the government accrue directly to the "people". Since the ultimate role of the business and household sectors is primarily one of being a producer of goods and services for the people, we can for simplicity assume as if all of the "defense benefits" accruing to the business sector are just being "passed through" to the people. The reason for this simplification will become clearer a little later.

The **household sector** has also certain assets, namely the **US Household Consumer Assets** (**UHCA**)**.** It generates its **US Household Total Products** (**UHTP**). It also receives products from both the business sector and the government sector, as already mentioned. Some of the total products are reinvested in its assets, namely the **US Household Products to Consumer Assets** (**UHPCA**). The remainder is delivered to the US resident people in the form of the **US Total Products to Consumption** (**UTPC**). Some of its assets are consumed in the production process, namely the **US Household Consumer Assets Loss** (**UHCAL**). As we already mentioned, the US resident people receive also the value of the US government defense activities.

There is a similar flow internally to the **ROW sector**, in a very simplified way. The ROW resident people provide their labor, the **ROW Total Production Labor** (**WTPL**) and receive the **ROW Total Products to Consumption** (**WTPC**). The ROW sector assets are organized in the **ROW Total Real Assets** (**WTRA**). The total production of the ROW sector is the **ROW Total Production** (**WTP**), part of which is reinvested as the **ROW Total Products to Assets** (**WTPA**. Some assets are consumed, as identified by the **ROW Total Assets Loss** (**WTAL**).

In the above descriptions we have assigned assets to various sectors on the basis of the sector's *use* of those assets, not necessarily their "ownership". Many apartment houses are owned by "businesses" which rent individual apartments to their occupants. However, it is the occupants who *use* the apartment, so in our terminology they "belong" to the household sector. The issue of "ownership" will be discussed in the following section.

We have allowed only for the USA Business sector to exchange goods and services with the ROW sector. In practice people (i.e. the "household sector") do get goods and services from the ROW, particularly when they are traveling. It is only for convenience that we are assuming that all the goods and services that the "people" get from the ROW sector, are actually obtained from the USA business sector, which in turn gets them from the ROW sector. The reason for this simplification is that the way statistical data is made available it would be difficult to separate the direct transactions of the people with the ROW, from the indirect ones through the USA business sector.

The flows of labor, goods and services discussed above are assumed to consist of “real things”, like hours of labor, wheat, clothes, automobiles, health services and the like. In practice, what we will be dealing with most of the time is the **value** of such labor, goods and services. In order to do that, we will have to decide how to assign a **relative value** to each “real thing”.

It should be pointed out that in our model the following assumptions have been made:

- each person is either a **US Resident** or a **ROW resident**;
- all labor utilized by the USA sectors derives from US resident persons and all labor utilized by the ROW sector derives from ROW resident persons;
- each asset is either a **US–located asset** (if it is associated with one of the USA sectors) or a **ROW-located asset**, if it is associated with the ROW sector;
- US-located assets are only utilized in **US Production** and ROW-located assets are only utilized in **ROW production**.

2.8 The Money Economy

In a modern industrialized society most of the economic activities consists of transactions in which one trades labor or gods or services for “money” or, more accurately for money instruments. We need to distinguish between “money” and “money instruments”.

As we mentioned in the previous sections, the role of the *real* economy is eventually to transform labor and the use of assets into goods and services. This means that the providers of labors and assets must eventually be “rewarded” with the goods and services that they “earned”. In such a process people will also exchange goods and services with one another. In an industrialized economy such exchanges cannot, in practice, be performed directly. What we do is to exchange labor or the use of assets or goods and services for “money instruments” and later we exchange money instruments for other labor, use of assets or goods and services. The role of *money instruments* is to *decouple* the activity of delivering one side of the “*real exchange*” from the activity of delivering the corresponding other side.

There is, however, an even more important aspect of the situation. Money instruments are expressed in **money units**. In order to decide *how many units* of money instruments we will need to exchange for any given item, a **price *at any given time*** will have to be associated with each item for which an “economic exchange” is possible. This de-facto establishes the ***current* relative market value** of the things we are *explicitly* “buying” or “selling”.

The use of money as a “unit of value” and of money instruments as the basis for economic transactions is certainly a giant convenience. It would be possible for the price of a cow to be US$ 2200 and the price of pigs to be $US 400. If we had to trade cows for pigs, we would have to get 5 and a half pig for our cow. This would be somewhat awkward. We can instead get only 5 pigs and keep US$ 200 to buy something else later.

However, there are two associated problems. The first one, well known to all of us, is the fact that if people hold on to their money instruments for a while, before converting them into other "real" things, the "money value" of the items they want to buy may be different than it was at the time they received the money instruments. This may be to their advantage or disadvantage, but in either case it creates an uncertainty. The second problem is associated with *international transactions*, in which the "money values" may be expressed in different "money units", creating a problem in establishing the "exchange value" of the different units.

The flows of the **money economy** are shown in Fig. 2.3. An important difference between Fig 2.2 and Fig. 2.3 is the presence of the **financial sector** as an additional element in the economic system. Many of the flows of financial instruments illustrated in Fig. 2.3 can be either *positive* or *negative*, i.e. they can actually flow in either one direction or the other. By definition, we assume that the flows are positive when they flow in the direction of the arrow in Fig. 2.3.

The financial sector will own no assets, either *real*, *money* or *equity*. It is responsible for "creating" equity assets in such a way that they have a zero total and distributing them to the other sectors. We define as the *total gross equity assets* of the USA business sector as the sum of the *real assets* (i.e. the *production assets* and the *inventory assets*) and the *money assets*. Since we assume that *all* "ultimate ownership" reside in the USA household USA government and ROW sectors, the *equity assets* of the USA business sector must *by definition* be set equal to the negative value of the *total gross equity assets*. This will imply that the total *net* assets of the USA business sector will always be exactly *zero*. A similar assumption holds for the *ROW business equity assets*. The role of the financial sector is to "create" the necessary additional business equity assets (possibly negative) to guarantee that such relations hold and to distribute the corresponding balancing equity assets (in some combination) to the USA Household sector and to the ROW Other Equity Assets. In addition to managing the creation of equity assets, the financial sector guarantees that the flows of "**interests and dividends**" are also in perfect balance. Each sector with a "negative" money or equity assets balance will typically pay money in interests or dividends. The financial sector guarantees that such payments are distributed to the sectors that have a positive balance, in such a way that the net flow of interests and dividends in and out of the financial sector is exactly *zero*.

The USA Household sector pays the **USA Household Payments to USA Business** (**uhvui**) to the USA business sector, as payment for the goods and services it receives. It also pays the **USA Household Taxes to USA Government** (**uhxug**). It receives from the USA business sector the **USA Business Wages to USA Household** (**uiwuh**) and from the USA Government sector the **USA Government Wages to USA Household** (**ugwuh**). In addition, it receives from the USA Government sector the **USA Government Transfers to USA Household** (**ugtuh**). The latter transfers consist primarily of the Social Security and Medicare *direct payments* to individuals. Finally, it receives from the Financial Sector the **USA Household Interests and Dividends** (**uhid**). It should be pointed out that the value of the variable "uhid" could be either positive or negative, as determined by the sign (positive or negative) of the USA Household total financial assets. The sectors also contributes the **USA Household Money Transfers to ROW** (**uhmtw**). Any net flow of money assets gets added to the USA Household Money Assets as the **USA Household Transfers to Money Assets** (**uhtma**). In addition to the money transfers just discussed, the USA household also receives a certain amount of **USA Household Transfers to Equity Assets** (**uhtqa**) from the Financial Sector.

The USA Business sector pays wages to the USA Household sector, as mentioned above; in addition it pays the **USA Business Taxes to USA Government** (**uixug**) and the **USA Business Money Transfers to ROW** (**uimtw**). It receives from the USA Household payments for the goods and services provided to it, as mentioned above; in addition it received form the USA Government sector the **USA Government Payments to USA Business** (**ugvui**) corresponding to the goods and services provided to the USA Government sector. It receives from the Financial sector the (possibly negative) **USA Business interests and Dividends** (**uiid).** The balance of the money flows is added to the USA Business Money Assets as the **USA Business Transfers to Money Assets (uitma**). Finally it receives from the Financial Sector its share (generally negative) of **USA Business Transfers to Equity Assets** (**uitqa**).

The USA Government sector receives taxes from the USA Household and Business sectors and makes payments to the USA Business Sector for the goods and services it receives. It pays wages and it also provides additional money transfers to the USA Household sector, as outlined earlier. It also provides the ROW sector with the **USA Government Money Transfers to ROW** (**ugmtw**), which consist primarily of grants to foreign government and agencies. It receives from the Financial Sector the **USA Government Interests and Dividends** (**ugid**). The balance of the money flows goes into the **USA Government Transfers to Money Assets** (**ugtma**)**.** It also receives its share of the equities as the **USA Government Transfers to Equity Assets** (**ugtqa**).

The ROW financial assets are organized as the **ROW Total Equity Assets** (**wtqa**) and the **ROW Total Money Assets** (**wtma**)**.** The ROW sector receives form the USA Government, USA Business and USA Household sectors the payments (possibly negative) outlined above. In addition it receives the **ROW Total Interests and Dividends** (**wtid**). The excess money flows into its **ROW Total Transfers to Money Assets** (**wttma**). It also receives its equity adjustment, the **ROW Total Transfers to Equity Assets** (**wttqa**).

The *equity assets* are "kept" in the "*equity accounts*", **ugqa**, **uiqa**, **uhqa** and **wtqa**. The overall total of such accounts must always be exactly zero, i.e. at all times

$$\mathbf{ugqa + uiqa + uhqa + wtqa = 0}$$

since all equities are created by the financial system in such a way as to maintain the validity of the above equation at all times. We assume also that a similar relationship holds for all *money assets*, i.e. at all times

$$\mathbf{ugma + uima + uhma + wtma = 0}$$

as we have already indicated earlier. Repeating our previous assertions, this implies that the governments must carry as a "debt" a value equivalent to the nominal value of all the currency notes and coins in circulation.

It is quite obvious that the real economy and the money economy are interrelated. We will discuss a little later certain specific relationships between the two, in particular the fact that the money economy will play an important role in the determination of the **value** of some of the real economy variables. However, we must appreciate the distinct role that they play. The *real* economy is what truly determines the (material) “standard of living” of the community as a whole. The *money* economy plays a major role in determining the specifics of the real economy, but it is only an *intermediary role*. It is perfectly possible to have two situations in which the real economies are exactly identical, while the money economies are different. From every practical point of view the two economies would be *equivalent*. It is important to keep in mind the distinction between the two economies. Otherwise, a number of issues may be obfuscated and one might reach inappropriate conclusions.

2.9 What is a “Nation”?

What are the assets that are “owned” by the USA or “owned” by the ROW? Before we answer that question we need to raise the issue of the concept of “nation”. The word “nation” is used with a variety of different meanings. Sometimes we use it to indicate a “set of people” and sometimes we use it to indicate a “physical territory”. It is hopeless to attempt to legislate what is the “appropriate” use of a word, since most people will keep using any way they like it. However, when attempting to establish a **logical consistent framework** for the analysis of any phenomena, it is necessary to agree on one definition, for the purpose of that framework.

When referring to “assets of the USA nation”, there are at least three reasonable definitions that we might use:

- assets owned by *people* who are *citizens* of the USA, as recognized by the USA Government;
- assets physically *present in the territory* controlled by the USA Government;
- assets owned by *people* physically *resident* in the territory controlled by the USA Government.

If every citizen of a nation would only be allowed to live in the territory of its own nation and not allowed to own any assets not physically located in such territory, all of the above definitions would be equivalent. However, in our world today the above does not apply. If this writer had its choice, he would favor the first one. In reality the other two are the ones used in practice in two different contexts, and for good reasons.

As we have already indicated, from the point of view of the “real assets”, it is the *physical location* that is the primary controlling element. Some real assets are *mobile*, like ships, automobiles and planes. In such cases, there is normally a form of *registration* that will determine the national association of the specific asset.

From the point of view of “ownership”, the way that the data is kept is on the basis of the ownership by the people who are *resident* in a given national territory. This is consistent with the fact that we have already mentioned, that the “USA Households” are designated to be those households that are resident in the USA, independently of the nationality of their members. Pragmatically, there is no realistic alternative.

Part of the confusion has to do with the distinction between **economic control** and **jurisdictional control**. The concept of asset ownership we implicitly referred to above is the kind which determines the **economic control**. Such control allows the owner to determine how the asset is used (within the law) *and* to accept whatever return on investment the specific asset will command. Today (2002) almost all nations allow, at least to some extent, that citizens of foreign nations can own assets in their own territory and that their own citizens can own assets in the territory of other nations.

The situation is quite different relative to **jurisdictional control**, i.e. the determination of which laws apply to the use of an asset. Almost all nations claim complete jurisdictional control on all assets physically residing (or registered) in their territory, independently of the nationality of their owners. The only negligible exceptions have to do with the "extraterritorial" status of certain assets officially associated with foreign governments.

Since the economic value of an asset is a function of its use, which is itself determined in part by the laws of the nation in whose territory the asset resides, there is an interaction between the concepts of economic and jurisdictional control. However, as long as all assets in a given nation are subject to the same laws, *independently of the nationality of their owners*, this fact is automatically reflected in their value, which supposedly would not be affected by changes in ownership.

What is the "production" of a nation?

Consider the following situation, P1. A plant is located in the US, but it is completely owned by Japanese investors, including the land on which the plant is located. The plant uses exclusively US labor. It uses only prime materials of negligible value (e.g. sand) to produce finished products, whose whole added value is therefore produced on US soil, with US labor, but with Japanese capital. Let's also assume that all of the products of the plant are sold inside the US. Of what nationality is the production of such plant? Before answering such question, let's look at a slightly different (and somewhat artificial) situation, P2. Assume that the same plant is actually located in Mexico, just across the US border. Assume again that all labor is US labor, which crosses into Mexico every morning and returns to the US in the evening. Of what nationality is the production of such plant?

There appear to be three factors to be taken into account, namely:

- the nationality of the **location of the plant**
- the nationality of the **workers**
- the nationality of the (owners of the) **capital**

One straightforward answer is that there are *two* productions involved here. The first consists of a **US-made service** (consisting exclusively of **labor**) that is being sold to Japanese interests, meaning that it is *exported* from the US to Japan (although it never physically goes to Japan). The second consists of **Japanese-made products** which are being *imported* into the US from Japan (even although they never were on Japanese soil). The total *net value* of *US imports* consists of the difference between the total value of the products and the value of the US labor, such difference being the value added attributable to the Japanese capital. In both situations P1 and P2, the location of the production facilities has no *economic* significance (although obviously the *political* significance may be great).

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However, this is not the accepted standard, partly because it would be very difficult to properly take into account all such situations. It is normally accepted that the **domestic production** of a nation consist of all the production which takes place *in its territory*, independent of any foreign ownership of the production assets. On the other hand, the **national production** is assumed to consist of all the goods and services produced by the **residents** of the nation in question. We will conform to such standard.

# 2.10 International Transactions

There is considerable confusion on the role of international transactions. We will address here only the issues that are relevant to our current analysis.

## 2.10.1 What is "International" Trade?

This is not as easy a question as it may seem. Consider the following two situations:

A. Mr. Nakamura arrives in the US from Japan with 10 Japanese-made automobiles, which he sells for $200,000. With the money received he purchases 10 new Detroit-made automobiles, with which he returns to Japan.

B. Mr. Nakamura arrives in the US from Japan with 10 Japanese-made automobiles, which he sells for $200,000. He then proceeds to go to Las Vegas where he loses all the money on the gaming tables, before returning to Japan.

In situation A there are clearly two international transactions. The two transactions balance each other from the point of view of trade balance, since an equal value of goods and services is exchanged.

Situation B is more confused. There are still two transactions, the first of which is the same as in situation A. However, the second transaction does not lead to anything being physically exported back to Japan. This will leave an imbalance in the international trade, with Japan looking as the net exporter. What happened in reality is that Mr. Nakamura "bought" a US-made service ("gambling") and consumed it on the spot. So there was a perfect exchange of goods and services between US and Japan, the same as in situation A.

From the point of view of the US, situation A implied an exchange of 10 automobiles for another 10 (supposedly equivalent) automobiles, with no gain or loss. In the case of situation B, the US gained 10 automobiles, and "paid" for it with a service that may have required only few hours of work by a Las Vegas card dealer. Clearly situation B, with its apparent negative trade balance, is to be preferred, from the point of view of the US.

This discrepancy is well known. There is always talk about the "invisible trade", which involves primarily the consumption of services in one country by citizens of a different country ("tourists"). Unfortunately it is almost impossible to have a detailed record of such transaction. Their contribution to the balance of trade can be at best approximately estimated. A most important issue can be clarified by considering another situation:
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C. Mr. Nakamura arrives in the US from Japan with 10 Japanese-made automobiles, which he sells for $200,000. With the money received he purchases a specially ordered, newly built home in Detroit. He then returns to Japan.

From the export/import accounting perspective, situation C, is analogous to situation B, i.e. the US had a net import of goods and services. This is obviously absurd. What is the difference between selling to a Japanese citizen 10 new automobiles, which he can bring back to Japan, and selling him an equal value new house that remains physically in the US? What is the difference in selling to a Japanese the services of Detroit automobile workers versus selling the services of Detroit construction workers? The same "equal value" trading has taken place in both cases. Actually, since in the second case the asset remains physically in the US, it would appear that the US is better off! Let's now consider another situation, namely

D. Mr. Nakamura arrives in the US from Japan with 10 Japanese-made automobiles, which he sells for $200,000. With the money he purchases an existing home in Detroit. He then returns to Japan. The seller of the existing home uses the returns to buy 10 Detroit-made automobiles for the use of his employees.

Situation D is a combination of A and C, with the net result that the US economy exchanged an existing 200,000 $ house for 10 new Japanese automobiles. Again, it would appear that situation D. is preferable to A., from the point of view of the US, since there has been a net increase in the assets physically located in the US itself.

2.10.2 Exports and Imports: Good or Bad?

In what follows we will always consider *exports* to be *USA* exports and obviously *imports* as *USA* imports. What are USA exports (as typically defined)? They are goods and services that have been produced within the US territory and are either physically transferred to the ROW' territory or are otherwise consumed by residents of the ROW. Imports are obviously the converse.

Upon reading the assertions of the "general press" on economics one would most certainly deduce that "exports are good (for the exporting country)" and "imports are bad". This is fundamentally wrong.

The typical export transaction involves the exchange of some USA product for a money payment. By definition, the values of the product and of the money payment are equal, at the time of the transaction. Nobody is gaining or losing. The same is true for an import transaction, in which the roles of the "seller" and of the "buyer" are opposite. In any given time interval it is extremely unlikely that exports and imports are in exact balance. Any small difference is obviously irrelevant. But what if there is a long-term imbalance? In the last 20 years of the 20th century the USA have had an excess of imports over exports. This means that the USA has had more goods and services to consume or invest that would have been the case if imports had not exceeded exports. Therefore the USA residents have enjoyed a higher level of consumption than otherwise possible, or the value of the assets physically located in the USA have grown more than otherwise possible. Either way, the USA has had a *short-term* windfall.

However, since the ROW did not send its goods and service for free, the USA must have transferred to the ROW an equivalent value in *financial instruments*. There is an important difference between *equity instruments* and *money instruments*. If the ROW has accepted equity instruments as payment for its excess imports into the USA, there are two possibilities:

1. The ROW was paid with ROW equities that the USA previously owned; in such a case the USA has experienced a decrease in its ROW assets.

2. The ROW was paid with USA equities, i.e. the ROW has essentially *acquired* assets that are physically located in the USA, as exemplified in examples C. and D. above; as we have already stated, this is just another form of “export”. The only problem is that it is accounted for differently.

A particularly interesting situation is the following:

E: Mr. Nakamura arrives in the US from Japan with a complete automobile production plant that it sets up somewhere in the US, under the ownership of a subsidiary corporation. He then returns to Japan with the shares of the subsidiary in his pocket.

The value of the industrial plant is obviously considered an import. The situation is conceptually similar to situation C, but in this case there is just a transfer of goods from one location to another, with no actual change of ownership. The USA is only affected as the physical host of the plant. However, as we will see in a later chapter, the presence of the plant on USA soil will be a boon to the USA economy since it will provide a net increase in the standard of living of the USA resident population.

If the USA has delivered to the ROW money instruments in payment for imports, the situation is a little different. Again, there are also here two cases:

1. The USA has returned to the ROW money instruments that signified a debt that the ROW owed to the USA; in this case the USA has experience a decrease in its credit toward the ROW.

2. The USA has actually incurred a new *debt* toward the ROW that must be repaid eventually with exports, either of the “standard” variety or through the transfer of ownership of assets.

In the latter case it may be arguable that the total net amount of outstanding debt should not be out of proportion to the total amount of assets owned. In other words, the amount of “leverage” that a nation allows itself should not be “excessive”.

2.10.3 Remittances

Apart from regular international “exchange” or *bilateral* transactions, in which different products of equal value are transferred between the USA and the ROW, there are certain other international transactions that could be called *unilateral* that play an important role.

Many nations issue **outright grants** to other nations, for political reasons. This is usually done by governments, but occasionally also by private organizations. In the case of the USA, it has provided extensive **foreign aid** since the end of World War II. These grants take the form of "money" that goes, let's say, from the USA to the ROW. The "money" then returns to the USA and it is exchanged for real goods and services that are *exported* to the ROW. The net flow of money is therefore essentially zero, but there is a net outflow of exported goods from the USA to the ROW (in the example given). These exports are real goods and services that are subtracted from the total production of the USA and therefore are a *net economic loss* for the USA residents. Obviously, there is an expectation of a corresponding *political gain* to be derived, but from the strictly economic point of view such exports are a net loss.

USA residents own ROW assets and ROW residents own USA assets, as we will discuss in more detail in the next chapter. Such assets "earn" a return. Such **returns on assets** can take the form of an increase in the value of the assets or it can be distributed to the owners in the form of interests and cash dividends. Depending on the value of the net balance of cross-owned assets this will mean a net flow of interests and dividends to the USA or to the ROW. In the early part of the time frame of this study the USA owned more assets abroad than the ROW owned in the USA. The net flow of interests and dividend money was therefore toward the USA. In the long run such money was returned to the ROW in exchange for real goods and services that were imported in the USA. Such "excess" imports were not paid with debt, but with the earnings of the USA owned assets in the ROW. In other words, they represented the "real" return of the USA investments in the ROW.

Some people move from the USA to the ROW and vice versa. While on foreign soil some of this people earn an income and some choose to send back part of their income to their "mother nation". Such **wage remittances** are analog to the above-mentioned return on assets. They represent, so to speak, a "return on labor". They eventually reappear as imports into the USA or exports to the ROW depending if there are more USA directed returns or vice versa.

2.10.4 Observations

Data about international transactions is very incomplete and often suspect, as we have already mentioned. We have tried to take into account the best available data. In some cases we have had to make certain assumptions in order to maintain data consistency. However, consistency is a necessary condition for accuracy, but not a sufficient one. We would warn the reader not to assume perfection in our assessment of international activities.

2.11 Total Real Assets

As we mentioned before, the real assets in the world represent the true wealth of the world. The *total* real assets that are associated with each of the USA economic sectors are the **US Government Real Assets** (**UGRA**), the **US Business Real Assets** (**UIR**A) and the **US Household Real Assets** (**UHRA**). They are obviously given by the relationships

US Government Real Assets =
US Government Civilian Assets +
US Government Defense Assets

US Business Real Assets =
US Business Production Assets +
US Business Inventory Assets

US Household Real Assets =
US Household Consumer Assets

The **US Total Real Assets** (**UTRA**) and are obviously the sum of the real assets associated with the USA Government, Business and Household sectors. In other words

US Total Real Assets =
US Government Real Assets +
US Business Real Assets +
US Household Real Assets

We have already summarized the **ROW Total Real Assets** (**WTRA**) in a single variable. The "grand total" **Worldwide Total Real Assets** (**WWTRA**) is obviously given by the sum

Worldwide Total Real Assets =
US Total Real Assets +
ROW Total Real Assets

We should reiterate that the **US Total Real Assets** represent the **real assets that are physically located in the USA**, while the **ROW Total Real Assets** represent **the real assets that are physically located in the ROW**.

2.12 Net Worth

The "net worth" of any individual or group of individuals is supposed to represent the total value of the *assets* that the individual or group owns *minus* the total value of the *liabilities* or *debt* that the individual or group might have. In our terminology, the **net worth** of any sector is the sum of the value of all its **real assets** and of all its **financial assets**. More precisely, the **US Government Net Worth** (**UGNW**) is given by

US Government Net Worth =
US Government Real Assets +
US Government Money Assets +
US Government Equity Assets

Similarly, the **US Business Net Worth** (**UBNW**), the **US Household Net Worth** (**UHNW**) and the **ROW Total Net Worth** (**WTNW**) are given by

US Business Net Worth =
US Business Real Assets +
US Business Money Assets +
US Business Equity Assets

US Household Net Worth =
US Household Real Assets +
US Household Money Assets +
US Household Equity Assets

Finally, the **US Total Net Worth** (**UTNW**) is given by

US Total Net Worth =
US Government Net Worth +
US Business Net Worth +
US Household Net Worth

that can also be expressed as

US Total Net Worth =
US Total Real Assets +
US Total Money Assets +
US Total Equity Assets

where we have defined

US Total Money Assets =
US Government Money Assets +
US Business Money Assets +
US Household Money Assets

and

US Total Equity Assets =
US Government Equity Assets +
US Business Equity Assets +
US Household Equity Assets

The **ROW Total Net Worth** (**WTNW**) is obviously given by

ROW Total Net Worth =
ROW Total Real Assets +
ROW Total Money Assets +
ROW Total Equity Assets

We should remember that the net total of all money assets must always be zero and the same must hold for all equity assets. Therefore

` ` `

US Total Money Assets = - ROW Total Money Assets
US Total Equity Assets = - ROW Total Equity Assets

The **Worldwide Total Net Worth** (**WWTNW**) is obviously defined as

Worldwide Total Net Worth =
US Total Net Worth +
ROW Total Net Worth

and obviously will satisfy the relation

Worldwide Total Net Worth = Worldwide Total Real Assets

reemphasizing again the fact that *financial assets* play only an intermediary role and that in the end only the real assets measure the total wealth of the world.

We have chosen, *by definition*, to assign all "true ownership" of US assets to the US Household and US Government sectors. This means that *by definition*

US Business Net Worth = 0

and therefore

US Business Equity Assets =
- (US Business Real Assets +
US Business Money Assets)

and

US Total Net Worth =
US Government Net Worth +
US Household Net Worth

2.13 The Measure of Production

One of the most important variables that people are concerned with is the "total production of the nation". If all of the world production would consist only of products that are either

- produced on US soil, with US owned assets and with labor provided by people that are US citizens and US residents or
- produced on ROW soil, with ROW owned assets and with labor provided by people that are ROW citizens and ROW residents;

there would be no problem. Obviously the reality is much more complex. If some products are produced on US soil, with ROW owned assets and by people who are ROW citizens but US residents, how should such products be labeled? As "US products" or as "ROW products"? There are many reasonable choices. However, all such choices should be "symmetric", i.e. if a certain combination of US and ROW activities would be labeled as "US products", the opposite situation, with the US and ROW roles reversed, should be counted as "ROW products".

A certain standard has been internationally accepted that allows us to divide all products as *either* part of the **US Gross Domestic Product** (**UGDP**) *or* of the **ROW Gross Domestic Product** (**WGDP**). We have implicitly used that standard. A somewhat different standard would allow us to divide products in the **US Gross National Product** (**UGNP**) and the **ROW Gross National Product** (**WGNP**). In the following we will deal only with the US variables, with the understanding that what is being said applies in an analog way to the ROW variables. Using the notation of Fig. 2.2, the US GDP is given by the total of the production of the three US sectors, namely,

US Gross Domestic Product =
US Government Total Products +
US Business Total Products +
US Household Total Products

The US GDP must eventually be "used" in some manner. In our model there are four such uses, namely

- the **US Total Products to Consumption (UTPC)**
- the **US Total Products to Assets (UTPA)**
- the **US Total (Net) Products to Exports (UTPX)**
- the **US Government Defense Services to US People (UGDSUP)**

Let's review the 4 components in more detail.

The "**US Total Products to Consumption (UTPC)**" component represents all those goods and services that the US people "consume", in order to derive sustenance and enjoyment. While the word "consumption" is standard in this context, it is actually a poor choice. The word "consumption" obviously means that "something is being consumed", but there is a negative connotation to the word, as if the meaning was that "something is being *destroyed*". In reality what we mean by "consumption" is that "something is being *used*". Of course, it is true that after something has been "consumed", that something no longer exist, but the *consumption* of the goods and services by people is what the goods and services were produced for in the first place. In our model that component is delivered to the **US Resident People** by the US Household sector.

The "**US Total Products to Assets (UTPA)**" component actually consists of the sum of 5 sub-components, namely

US Total Products to Assets =
US Government Civilian Products to Assets +
US Government Defense Products to Assets +
US Business Products to Production Assets +
US Business Products to Inventory Assets +
US Household Products to Consumer Assets

` ` `

as outlined in the flow described in Fig. 3.2. It corresponds to the total (gross) additions to assets generated by the three US economic sectors. As we shall see later on, this component plays a dominant role in determining the value of the US GDP *in the future*.

The "**US Total (Net) Products to Exports (UTPX)**" component consists of the *difference* between the products exported by the US and those imported by the US, namely

US Total (Net) Products to Exports =
US Business Products to ROW –
ROW Products to US Business

We recall that we have assumed for convenience that all goods and services transactions between the USA and the ROW actually occur through the USA Business sector. It should be noted that the value of the UTPX component is *positive* if the US exports more goods and services than it imports and *negative* otherwise.

The "**US Government Defense Services to US People (UGDSUP)**" component is a special one. The amount of money that people decide to spend on defense is a form of "insurance". It is done in the expectation that it will prevent external forces from interfering with the people's lives and properties. However, it is a form of insurance whose "payoff" is never actually visible. In peacetime, the *benefits* deriving from the US Government defense activities may be important in *political terms*, but are not visible in *economic terms*. We will return on this issue a little later. On the basis if the above, we can therefore express the US Gross Domestic Product as follows:

US Gross Domestic Product =
US Total Products to Consumption +
US Total Products to Assets +
US Total (Net) Products to Exports +
US Government Defense Services to US People

As implicit in its definition, the GDP refers to the "gross" production. Such measure is useful for some issues, but it is very misleading n other contexts. Suppose you have two identical housing lots, one empty and one with a house worth US$ 200,000. Suppose that on the empty lot you build a house worth US$ 500,000 and suppose that on the second lot you destroy the old house and build also a new house worth US$ 500,000. From the point of view of the "gross" domestic product, the two activities are identical, both adding US$ 500,000 to the GDP. However, the two situations are quite different from the point of view of the change in the wealth of the nation.

We can define the "**US Total Asset Loss (UTAL)**" as the sum of all assets losses incurred by the US economic sectors, namely,

US Total Asset Loss =
US Government Civilian Asset Loss +
US Government Defense Asset Loss +
US Business Production Asset Loss +
US Household Consumer Asset Loss

What we have called "asset loss" is often referred to in standard terminology as "asset consumption". We have chosen not to use that term, since we want to keep the term "consumption" to refer exclusively to the "people's consumption" of goods and services. As we mentioned earlier, such consumption is truly a *use* of goods and services and is therefore directly related to the 'standard of living". On the other hand, "asset *consumption*" is just an "asset *loss*", with no associated "benefit" deriving from it.

We are now in the position to define the **US Net Domestic Product** (**UNDP**) as

US Net Domestic Product = US Gross Domestic Product – US Total Asset Loss

We can obviously express the US Net Domestic product also as follows

US Net Domestic Product =
US Total products to Consumption +
US Net Products to Assets +
US Total (Net) Products to Exports +
US Government Defense Services to US People

where we have defined the **US Net Products to Assets (UNPA)** as

US Net Products to Assets =
US Total Products to Assets –
US Total Assets Loss

By definition the **US Net Products to Assets** measures the net increase in the **domestic assets** of the USA.

Chapter 3

The Value of Money and the Rate of Interest

As we have already mentioned, in order to compare economic data at different times we must take into account the change in the **value of money.** Changes in the value of money also affect the **interest rates** charged borrowers. We will investigate those issues in this chapter.

3.1 The Value of Money

It is sometime surprising how often in the American press prices at different times are compared without taking into account the changes in the value of money. Often we hear cries about how expensive something is, compared with the “good old days”. In almost all cases, the *numerically higher* cost of an item actually corresponds to a *lower* cost, when measured in “constant dollars”.

There are two real difficulties:

- First of all, it is not always obvious what is the most appropriate algorithm for normalizing the value of money.
- Second, it is not always possible to guarantee that the goods or services under consideration at different times have truly the *same intrinsic characteristics*.

The most commonly used metric for price normalization, the **Consumer Price Index (CPI)**, suffers from an inability to properly take into account substitute goods. So it tends often to overstate the level of price increases, since it does not accounts for the consumer shifts from more expensive to cheaper alternate goods.

The problem is the following. Assume that at a given time the average person consumes 1 pound of beef and 1 pound of chicken per week. Assume that the price of beef is 10 $/pound and the price of chicken is 5$/pound, for an average expense of 15 $/week. Now assume that the following year the price of beef is 15$/pound while the price of chicken is still 5 $/lb. The cost of the same mix of products would now be 20 $/week, for an increase of 33%. However, because of the increase in the price of beef, let’s assume that now the average person consumes 1/2 pound of beef and 1 and 1/2 pound of chicken. The cost of the new mix would be 15 $/week, with no increase at all in cost. On the other hand, since the people are willing to pay more for beef, it must mean that beef is a more attractive product. Therefore the new mix, which includes less beef, is "inferior" in some respects. It is not clear what the most appropriate choice of correction is, namely the one corresponding to a fixed quality of goods or the one corresponding to the actual purchasing choice of the people.

The agency responsible for the determination of the CPI is the US Department of Labor. When the CPI increases, the estimate of the "real wages" paid to employee obviously will decrease. This will give support to labor demands for increases in "nominal wages". Since organized labor is the major constituency that the US Department of Labor recognizes, it tends to bias the Department toward overstating the increases in CPI.

The Bureau of Economic Analysis (BEA) of the Department of Commerce is responsible for the estimation of the "real value" of the GDP, through the estimation of the "**GDP deflator**". It is generally advantageous to Administration to be able to show that the GDP is "doing well". Accepting the CPI measure as the measurement of the "value of money" would depress the value of the GDP in real terms. Therefore the methodology used by the BEA is quite different. It is based on the so-called "Fisher chain indices". We will discuss the methodology in some detail in Appendix A. Here it will suffice to state that the methodology does indeed handle the problem of goods substitution mentioned earlier, albeit in a "pragmatic" way, not based on a coherent approach to the issue. Furthermore, it carries with it a number of other problems, as will be discussed in Appendix A.

We will base our analysis on a completely different approach, also to be discussed in Appendix A, which is based on a plausible model of consumer behavior. The cost of the new approach is the much higher degree of computational complexity. We will here present some of the final results.

Fig 3.1 shows the **value of money** in the years 1950-2010 expressed in year 2010 US$. In other words, for example, one 1950 dollar was worth approximately 8 year 2010 dollars, while in 1980 a US$ was worth about 2.3 2010 dollars. A different way to look at the same data is shown in Fig. 3.2 which shows the **price index** in 2010 US$. It shows that, on the average, what cost 1 US$ in 2010 would have cost about 82 cents in the year 2000, 43 cents in the year 1980 and about 12 cents in the year 1950.

Fig. 3.3 shows the (annualized) quarter-to-quarter **inflation rate**, based on the curve of Fig. 3.1. In the 50's and 60's the inflation rate was between 2% and 4%. In the 70's it jumped upward to the 6%-10% range. It then came back to the 2%-4% range in the late 80's and in the 90's. Notice that the somewhat high rate of inflation in 1950-1951 may be a result of an anomaly in the data and may not be real.

From the point of view of economic analysis it would be nice if the inflation rate would be zero, i.e. if the value of money would stay constant. In practice, an inflation rate of about 2% is often considered "unavoidable".

In the financial world there are many different interests rate that play different roles in the economy. We have chosen the so-called **prime rate** as an overall representative of the spectrum of interest rates. Fig. 3.4 shows the prime rate of interest, together with the inflation rate already shown in Fig. 3.3. We have put both curves on the same graph since in a general way the prime rate “ought” to be approximately equal to a constant “real” rate of interest plus the inflation rate.

Fig. 3.5 shows the “r**eal” interest rate**, i.e. the actual difference between the prime rate and the inflation rate. Fig. 3.5 shows an interesting pattern. In the years before 1970, the “real” interest rate hovered around the 2% level (with major variations). Then, during the years of high inflation rate, the “real” interest rate went as low as to becoming negative and then it reached extremely high levels (up to 10%) in the late 70’s and the early 80’s. Since then the “real” interest rate as become less erratic, but it hovered around a noticeably higher levels (approximately 5.5%) than those of the 50’s and 60’s. In the most recent years the real interest rate has again shown larger swings.

Chapter 4

Basic Demographics

One of the major underlying factors in the economy of the USA is the size and composition of its population. In this chapter we will review some of the most important elements.

4.1 The Total Population

For pragmatic reasons, most economic data in the USA is collected on the basis of its relevance to the **resident population**. This is defined as all of the people who are *long-term resident* in the USA territory, independently from their nationality. This means that all foreigners who have their primary residence in the USA are included, while all USA citizens who are permanently living abroad are excluded. In particular, the resident population excludes members of the Armed Forces living abroad. The **total population** includes members of the Armed Forces living abroad, while still excluding civilian USA citizens who are residents of a foreign country.

Every 10 years the Census Bureau performs an official census that establishes the official population numbers for that year and the details of its composition. There have been a number of criticisms about the methodologies used by the Census Bureau, mainly claiming that members of certain minorities are being undercounted. While there may be some truth to the criticisms, one must take into account the factor that many of such criticisms are politically motivated. However, one issue that is certainly significant is the counting of *illegal long-term residents*. Estimates about their number are obviously unreliable. However, they are almost certainly a sizable number. How many are being counted as *legal residents* (since the Census Bureau does not actually investigate the issue) and how many are *not counted* at all is open to question.

For non-census years, a number of methodologies are used to determine the appropriate values. While such *estimates* are obviously less reliable then the 10-year census data, they are probably reasonably accurate for intervening years between known census years. They may not be very reliable as extrapolations to years beyond the last census year. For example, the estimates published in 1999 for the population in the year 2000 gave a range of between 271,237,000 and 278,129,000. The published official data for year 2000 census was 281,491,906, approximately 1% higher than the highest estimate.

For our analysis these uncertainties are not very significant, if we believe that the rate of error has remained approximately the same over the years. Under this assumption, the absolute values of some of the quantities we will compute may be slightly inaccurate, but the year-to-year relative data can be considered reasonably reliable.

Fig. 4.1 gives the year–to-year change in population from 1950 to 2010. The growth rate has decreased dramatically from 1950 to 1970, from almost 2% to approximately 1%. It has remained essentially flat since then. The 1% total growth rate is due for about 2/3 to internal growth (i.e. births minus deaths) and for about 1/3 to net migration. The growth rate in the period 1990-2000 is shown as a very straight line. This is because the values have been derived by assuming a constant rate of growth in that period, since the actual data provided by the Census Bureau were grossly incorrect.

Fig. 4.2 shows the changes in age composition of the population from 1950 to 2010. We need to explain the definition of **youth**, **working-age** and **senior** components. It has been tradition to define the "working age population" as everybody 16 years old or older. At the beginning of the century a large segment of the population lived on farms. This meant that many people started contributing to the farm operation quite early in life. By the time they were 16 years old, many young people were significant contributors to the economy. At the opposite end of the age scale, older people would also contribute to the farm operation, at least since they were physically capable of doing so. When they were not, most of them would die. In our current society the situation is quite different. Although some teenagers do part time work (at McDonald's and similar establishments, for example), the majority of them enter the working population after they graduate from high school, i.e. after they are 18 years of age. At the other end of the scale, the official age of "retirement" has been set at around 65 years. Some people retire earlier, but, with the considerable improvements in life expectancy and old age health, many choose to retire later than that. However, many of the older people now live long lives beyond the ages of 65, therefore remaining a consuming element of society.

We have chosen to define the **working age population** in a somewhat complex way. It includes **everybody between the ages of 20 and 64**. In addition it includes:

20% of people aged 16
40% of people aged 17
60% of people aged 18
80% of people aged 19

90% of people aged 65
80% of people aged 66
70% of people aged 67
60% of people aged 68
50% of people aged 69
40% of people aged 70
30% of people aged 71
20% of people aged 72
10% of people aged 73

All people younger than the working age population are defined as being the **youths** population; all people older than the working age population are defined as being the **seniors** population. Such definitions are clearly arbitrary. They attempt to provide a simple approximation to the identification of people who are, on the average, capable of "working" from both the physical and mental point of view. For simplicity of terminology we will refer to "working age people" as **adults**.

The percentage of youths started at about 30% in 1950 and increased up to the mid 60's. At that point it started decreasing substantially, apparently reaching a recently stable value of about 25%. At the other end, the percentage of seniors has been increasingly steadily from about 5% in 1950 to almost 10% in 2000. It has then stabilized around that value. Interestingly enough the percentage of adults started at about 65%, went down to about 59% in the mid 60's and then went back up to about 65% again, where it appears to have stabilized.

4.2 The Employed Population

One of the most important economic variables is the number of people that are **actually employed**. We should clarify that by "employed" we mean that the person is engaged in a *formally acknowledged economic activity* from which it derives explicit money wages. This includes not only people employed by others, but also people *actively* engaged in business, either as proprietors or in a partnership. For simplicity we will refer to all these people as **employed persons**.

There are two distinct ways to account for the number of employed persons in a given period of time. The first is to count all the persons that have been employed for at least part of the time in that period. The second is to count the "full time equivalent (FTE)" number of employed persons, i.e. weighing the contribution of each person by the percentage of time actually worked. It is not clear which is the most appropriate number to use in a systematic analysis. From the point of view of assessing people's participation in the economic system, the first measure would appear to be the most appropriate. From the point of view of assessing productivity, the second measure would appear to be preferable. We will refer to the first count as **total employed persons** and to second count as **FTE employed persons**.

Fig. 4.3 shows both variables as ratios to the total number of adults, i.e. working-age persons. Fig. 4.4 shows two interesting ratios, i.e. the ratio of FTE employed to total employed and the percentage of women workers. Although precise figures are not available, there is some anecdotic evidence that women are part time workers more often than men. The data would appear to partially confirm this, but the correlation is far from being a solid one.

In Fig. 4.5 we compare the evolution of the workforce participation ratios for men and women. The change has been impressive. In the last 60 years there has been a substantial increase in the percentage of employed women. In the 60's and 70's this apparently led to a reduction in the percentage of employed men. Clearly, the most marginally competent male employees were being replaced by the more skilled, but hitherto unutilized, females. Since the mid 80's, the percentage of employed men appears to have stabilized, while the percentage of employed women has continued to increase up to year 2000.

Starting around the year 2000 there appears to be the beginning of a decrease in the participation ratios of both men and women. The period between 2004 and 2008 was one of economic expansion. During such periods we normally see an increase in the overall workforce participation ratios. We will come back to this issue in a later chapter.

There are a number of problems with the accounting of "employed" and "non-employed" people. First of all is the issue of how to account for people who do not work the whole year because of the *intrinsic nature of their work*, like crab fishermen in Alaska whose "work season" may only be a few months long. Also, our definition of "adults" did not take into account the number of people in the appropriate age brackets who may be *totally incapable* of working. This would include, among others, people with incurable health problems, people with severe mental problems and people who are in prison. Additionally, the account of "employed" people almost certainly does not include all the people who are gainfully employed in *illegal activities*, without the benefit of a "cover job". The latter problem certainly distorts the employment figures for the younger people, among whom we are likely to find most of the prostitutes (of both sexes) and of street level drug dealers. These factors are important because they indicate that the "absolute ceiling" for "official" employment may be considerably lower than the 100% of "working age people" than one might expect. It is not unlikely that for men the practical ceiling for "full time equivalent" employment is not too far away from approximately 75%. Of course, a redefinition of "working age" people that would allow for a larger inclusion of older people would change the numbers. It is also reasonable to suppose that although the percentage of employed women may continue to approach that of employed men, it may never reach quite the same level. This is due to the fact that women are biologically more likely to take some time off work in association with their pregnancies and the care of infants in the very first months of life. If we assume that, on average, each woman has two children and may take six to 12 months off work for each child, this would lead to an average differential of about 2% to 5% in the working ratios of male and females. However, since the female adult population larger than the male's, it is likely that in the not too distant future the total number of female workers may exceed that of male workers.

4.3 "Unemployment"

One of the most often quoted figures is the rate of "unemployment". This is a most questionable figure. It is based on the number of people who do not have a job but are "looking for a job". The problem is that it may include seasonal workers, who are "out of a job" outside their own working season, but have no real possibility or interest for an alternative job. Also, it may include people who may be looking for a job, but either have no marketable skills or may be "overqualified" for the job openings actually available. If a person who is the sole breadwinner in a family gets fired, we may have that both that person and the person's spouse start looking for a job. In other words, we have one less worker and two more "unemployed". At the other extreme, a person fired from a job may "drop out" of the job market for various reasons, therefore not appearing as an increase in unemployment.

In Fig. 4.6 we show the **unemployed** percentage. Our percentage of "unemployed" is *not* what is normally published as the "unemployment rate". The official unemployment rate is measured relative to the "*labor force*", i.e. to the sum of employed and unemployed people. What we are showing is the percentage relative to the **working age people**, so that the "employed" and "unemployed" figures are comparable. In addition we show also the **labor force**, as defined above. Note that the "unemployed" data is plotted according to the right hand vertical axis, while the "employed" and the "labor force" are plotted relative to the left hand axis, which is offset, so that the bottom of the scale reads "50%". The concept of the "labor force" is that it should be measuring the total rate of "employable" people. Unfortunately, for the reasons mentioned earlier it is not a good measure. Obviously the changes in the rate of unemployment correspond to the changes, in the opposite direction, of the rate of employment. Major changes in the employment rate are obviously associated with economic cycles. Employment goes up during expansions and down during contractions. One interesting fact is that the "labor force", i.e. is also affected by the economic cycle. When employment decreases, also the total labor force tends to decrease. When employment increases, the labor force increases.

In Fig. 4.7 we have plotted the changes in *employment rate* together with the changes in the "official" *unemployment rate*, with the latter having been changed in sign, so that they are more directly comparable.

The raw data is very "jumpy", so we have smoothed the data somewhat. Obviously the two variables track each other most of the times. The point we wanted to note, however, is that often, during an expansionary period, when the employment rate goes up, the unemployment rate fails to follow or decreases more slowly. This may be due to a combination of two factors. The first is that the people being hired in an expansion may not be the same people that were fired earlier, but "new workers", not members of the "labor force" before then. The second is that as workers are being hired away from the pool of unemployed, others become encouraged and enter or re-enter the labor force, as temporarily "unemployed". Fundamentally we believe that the official unemployment data is of little use and it is more appropriate to focus on the *employment* data, which is not subject to spurious "psychological" effects.

4.4 The Dependents Ratio

The same data that we have been showing in the previous graphs can be presented in a different way to illustrate an issue that is often discussed in the press. For every "gainfully employed" person there are on the average a number of *economical dependents*, who must be "economically supported" by the employee. These "dependents" come in three groups, namely the *youths*, the *seniors* and the *non-employed adults*.

Fig 4.8 shows the average number of each group, per full time equivalent employee, over the 60 year period. Clearly the number of youths per employee has dramatically decreased because of the combined effect of the decrease in the percentage of youths and the increase in the percentage of employees. The number of seniors per employee has remained relatively stable, after a minor increase in the 50's, since the increase in the percentage of seniors has been matched by the increase in the percentage of employees. More noticeable is the change in the ratio of non-employed adults to employees. The obvious reason is that every increase in the percentage of employees, simultaneously *decreases the numerator* of the ratio (the number of non-employed) and *increases the denominator* (the number of employees). This “double effect” is also responsible for the apparently large swings in the ratio, due to changes in overall employment rate during the vagaries of the economic cycle.

The graph shows that, in aggregate, the total number of dependents per employee increased somewhat during the 50’s, but has since then decreased considerably and by the year 2010 is about 2/3 of what it was in 1960. We will return to this issue later on, but we wanted to make the point that the problem of increased longevity, with its associated increased ratio of seniors, is not a significant one.

It should be noted that the increase in the dependent ratio after 2007 has been due exclusively to the increase in *non-employed adults*, due to the increase in unemployment.

4.5 Summary

In this chapter we have outlined some of the major historical demographic trends in the last 60 years. The most important ones are the significant increase in women participation in the work force and the fact that such increase has led somewhat to a substitution of men with women, but to more significantly to a major overall increase in the employment rate.

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# Chapter 5

# History of Key Economic Variables

In this chapter we will review the history of the key economic variables, within the structure of our model. We repeat here that all data presented below, when expressed in dollars, it is expressed in **equivalent year 2010 dollars**. The normalization is performed using the data shown in Fig. 3.1.

## 5.1 The Gross Domestic Product (GDP)

In Chapter 2 we have outlined the structural composition of the **Gross Domestic Product (GDP)**. The GDP measures the production of goods and services that physically takes place *within the boundary of the territory of the US*. Suppose that there is a plant, located in the US, but that is fully owned by foreign interests. Suppose that the plant uses exclusively US labor. Assume that it only uses prime materials of negligible value to produce finished products that are exported to the rest of the world (ROW). Such production would be considered part of the US GDP. Now suppose that the same plant would be actually physically located in Mexico, just across the US border and that again all of the labor would be US labor, which crosses into Mexico every morning and returns to the US every night. Now the production would be considered part of the Mexico GDP. We mention this example to show that the "conventional" definition of the GDP may sometimes lead to strange evaluations.

We can now look at the history of the GDP, as shown in Fig. 5.1. The GDP has grown by about a factor of 7 from 1950 to 2010. However, the *total* GDP can be a misleading figure. We believe that a more appropriated measure is the **GDP/person**. The history of that variable is shown in Fig. 5.2. Fig. 5.2 shows that the GDP/Person has increased by a little more than a factor of 3 over the 60-year period.

Quite obviously the rate of growth has not been constant. Fig. 5.3 shows the percentage growth on a yearly basis, i.e. measured by comparing each quarter with the corresponding quarter of the previous year. There are a couple of observations to be made regarding the data in Fig. 5.3. The first one is about the abnormally high rate of growth for the first year in the graph, namely the year 1951. It is very likely that such growth was not real, but is an artifact due to partially inaccurate data. The second one is about the fact that there appears to be a two-year oscillation in the value of the growth rate. This is also probably due to partially inaccurate data. If for some reason the estimate of the GDP for any one year is somewhat lower that its actual value, this will result in an *underestimate* of the growth for that year. If the following year the error is corrected, that will result in an *overestimate* of the
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growth for that year. The linear trend line shown indicates that the *average* of the growth rate is showing a decrease over the years, from approximately 2.4% in 1950 to about 1.8% at the end of 2010. Interestingly enough, the *average* growth rate has been rather steady at 2% from 1950 to 1990, but with considerable year-to-year oscillations. From 1990 to the present the oscillations have been smaller, but the average rate of growth has deteriorated.

The latest economic downturn, in the 2008-2009 period has led to a negative growth of about 4%, the worst since the mid 80's, and it is the main reason for the negative trend.

The Bureau of Economic Analysis (BEA), an agency of the US Department of Commerce, is responsible for collecting the relevant data and then estimating the value of the GDP. From time to time, the BEA changes some aspects of its methodology. In order to maintain comparability of data, the changes are retroactively applied to the older data. However, the older data might not contain the required information. In such cases the process is likely to produce some distortions. In our judgment, the value of the GDP estimated for any given year cannot be considered more accurate that within 1-2%. This suggests that small short term variations should be considered unreliable and should be ignored.

The total GDP results from the activities of the three primary economic sectors, namely the **Government**, **Business** and **Household** sectors. Their relative contribution is shown in Fig. 5.4. Quite clearly the business sector is by far the major contributor, with approximately 76% of the total. Each of the Government and Household sectors represents approximately 10%-12% of the total. Over the 60 year period the percent contributions of the three sectors have remained remarkably constant.

5.2 The Gross Domestic Activity (GDA)

The GDP is the most commonly referenced aggregate economic variable. If there would be no international transactions or if the value of exports and imports would exactly match, the GDP would provide an accurate measure of the economic activity of the nation. However, the above does not apply, at least in general. If exports exceed imports, it means that more products "go out" than "come in" and therefore the goods and services that remain available to the residents of the nation are, in aggregate, less than the goods and services that were produced. Obviously, the opposite happens if imports exceed exports. We will define the **Gross Domestic Activity (GDA)** as

Gross Domestic Activity = Gross Domestic Product –
Exports + Imports

The GDA more accurately measures the sum total of goods and services that were *available to the USA residents*.

Fig. 5.5 shows the amounts of Exports and Imports over the years and the **net** exports, i.e. the difference between Exports and Imports, which in recent years has been always negative. The average between Exports and Imports, is a measure of the overall **products trade activity** and is shown in Fig 5.6, relative to GDP. It is quite clear that the importance of trade activity has grown considerably over the years, representing currently about 14% of GDP.

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The difference between the GDA and the GDP (i.e. the net excess of imports over exports) is shown in Fig. 5.7 as a fraction of the GDP. Up to the mid 70's the difference was relatively small and, mostly, was negative, i.e. there was an excess of exports over imports. Starting in the mid 70's a definite trend toward an excess of imports over exports was established (triggered in part by the sudden rise in oil prices). There have been some major oscillations, but for now the trend appears to be clearly established.

The reason for introducing the new variable is that certain measures that we want to discuss below cannot be correctly expressed relative to the GDP, but must be expressed in relation to the GDA. Although the difference is not large, there would be logical discrepancies if we did otherwise.

The value of the GDA is shown in Fig. 5.8, and the value of the GDA per person shown in Fig. 5.9. One interesting perspective is shown in Fig. 5.10. There we show the percentage of the GDA that is attributable to the Government **defense** and **civilian** activities, together with the total of the two. The total Government activity has been decreasing slowly over the years, form a high of 24% in the mid 50's to a low of about 17% in the year 2000, but it has increased again after that, partly because of the reaction, both domestically and abroad, to the events of September 11, 2001. However, the composition has changed drastically over the years. In the 50's, the defense and civilian components were approximately equal, each being approximately 10% of GDA. Starting in the mid 50's, the defense component has decreased significantly so that in the year 2000 less than 4% of the GDA was devoted to defense. The war in Iraq has been responsible for the later increase of that component up to 5%. In the meantime the civilian Government component of the GDA had risen up to 16% in the mid 70's and later decreased somewhat to about 14%. It should be noted that the overall Government contribution has decreased in the 90's to levels not seen since the early 50's. However, such reductions are almost completely due to reductions in the defense component (at least until 2003), with the civilian component remaining approximately constant. It may be noted that while the Government GDP has been only about 11% of the total GDP, the Government contribution to the GDA has been in the range of 20%. The apparent discrepancy is explained by the fact that the *Government GDP* consists of all goods and services that are produced *directly* by the Government, through its own employees. The *total Government activities* include also all the goods and services that the Government *purchases* from the Business sector and then delivers to the Household sector.

In Fig. 5.11 we show another view of the GDA, as determined by the *ultimate use* of the goods and services. The partition between *total consumption* (civilian plus defense) and *gross total investment* (civilian plus defense) has remained reasonably constant at the 80%-20% level. However, both the defense consumption and the defense investments have been decreasing considerably over the years, as already mentioned earlier.

## 5.3 The Material Standard of Living

Too often people equate the **standard of living** with the GDP/Person. This is quite wrong. The *current* standard of living of a community is determined not by what it *produces*, but by what it *consumes*. To put it in simpler terms, a person's standard of living is not determined
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by what that person *earns*, but by what that person *spends* for goods and services (or what somebody else spends to provide that person with goods and services).

The **total consumption** is the sum of the **civilian consumption** and the **defense consumption**. The civilian consumption includes all the goods and services that the people *directly* enjoy. The role of the defense consumption is essentially one of *insurance*. Its value consists in providing the people with a certain degree of "peace of mind" that they will be able to continue to enjoy their goods and services, without foreign interference. If we take a very narrow view of the concept of "standard of living", we would conclude that the **material standard of living** is essentially determined by the **civilian consumption per person.**

Fig. 5.12 shows the Civilian Consumption per person. The curve shows an average compound growth rate of about 2%, slightly higher than the growth of the GDP/person. The main reasons are the decrease in the defense component of the USA economy and the excess of imports over exports. Fig 5.13 shows the yearly percent change, on a year-to-year basis, i.e. comparing each quarter with the same quarter the previous year. It shows that there were very few years in which the civilian consumption per person actually decreased from a year earlier, particularly after the first ten years. The data in Fig 5.13 is very jumpy. Figure 5.14 shows the same data on a 4 year basis, i.e. comparing each quarter with the same quarter four year earlier. On that basis the average growth for a four year period is shown to have been slowly increasing from 8% to about 10%. There have been only two quarters in 1982 in which the growth was negative. Even in 2009-2010 the average civilian consumption per person was higher than 4 year earlier, albeit by only 2%.

It is interesting to note that the rate of year-to-year change in civilian consumption per person has been increasingly slowly from about 2% to 2.3%, while the corresponding variable for the GDP/person has been decreasing from 2.4% to about 1.8%. Consumption habits tend to change more slowly and larger increases or decreases in the GDP tend to lead to corresponding larger positive or negative changes in the rate of *investment*.

5.4 Gross vs. Net Investments

The overall **gross (domestic) investments** have been about 20% of GDA, as shown in Fig. 5.15. However, such figure gives an incomplete picture of the situation. A considerable portion of the *gross* investments goes toward the replacement of the assets that have been "consumed" during the production process. We refer to such "consumption" of assets as **assets loss,** so as not to confuse it with "true consumption" by "consumers". Unfortunately there is a considerable difficulty in assessing the value of the assets loss, since the "depreciation" information provided by business is often an artificial figure.

Fig. 5.15 shows the gross assets investments and the assets loss as percentage of the GDA. Notice that the rate of assets loss, relative to GDA, increased in the 70's from less than 10% to about 11%. This probably reflects in part the increased fraction of assets that can be considered "high tech" and which may be subject to somewhat faster obsolescence. At the same time the rate of gross investment has been decreasing.

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Fig. 5.16 shows the **Net Investments**, i.e. the difference between the gross investments and the assets loss. The rate of net investments has been decreasing from a high of about 12% in the 50's to about 6% in 2010. There was a considerable decrease starting in the mid 80's, down to a level of only 6% in the early 90's. Since then there has been renewed growth for a while, but a new period of decrease, exacerbated by the latest downturn in the economy has brought the investment rate down to 2% again toward the end of 2009.

It is interesting to compare the rate of investment for the government sector and the **private** sector (i.e. the combination of the business and household sectors). Fig. 5.17 shows the data as a fraction of GDA. It is interesting to note that the Government rate of investments tends to be much more stable than the private rate. This is quite understandable, since the government budgets are often set a priori, without much consideration given to current economic factors, while the private sector reacts more quickly to changes in the economic environment. However, the allocation of the government investments has changed considerably over the years. Fig. 5.18 shows the rates for the defense and the civilian sectors of the government net investments. In the civilian area there has been an increase up to the mid 60's, followed by a decrease to about 2% where it appears to have stabilized. The defense component has gone instead through major upheavals. From about 2% in the early 50's (during the Korean war) there has been a drastic decline during the 60's (even during the Vietnam war) until it actually became *negative* in the 70's. The investment rate went back up in the 80's and then went negative again in the 90's. There has been a slight increase in the last few years.

Fig. 5.19 shows the partition of the private investments between the business and the household sectors. The business component of the private net investment has maintained a reasonably constant average of about 4.5%, with major short term variations, up until the year 2000. After that there has been a substantial decrease to about 3%. The household component has also shown considerably short term variations, but has stayed close to about 2.5%. However, the housing crisis of 2008-09 has brought the household investment rate down to negative values.

# 5.5 USA Domestic Assets

The **USA Domestic Assets** are those physically located within the USA territory. The net changes in those assets are represented exactly by the net investment discussed in the previous sections. The overall asset total over the 60 year period is given in Fig. 5.20. Fig 5.21 and Fig. 5.22 show the repartition of those assets among the three economic sectors, in absolute and as a relative fraction. The repartition has changed considerably from the 1950's to the 1990's. The household component has surpassed the government component, while the business component has increased somewhat. Since the 90's the relative proportions appear to have somewhat stabilized.

# 5.6 International Transactions

In Fig. 5.5 we have shown the **balance of trade in goods and services** in the form of the excess of imports over exports. However, as we have discussed in Chapter 2, there is
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also international trade in **assets (equities and money).** The balance of trade in goods and services, equities and money (as USA exports minus USA imports) is shown in Fig. 5.23. Fig. 5.24 shows the *net USA imports* together with the sum of the *net USA equities and money exports.* Note that the curves are very close to each other. This is obvious, since the excess of imports must be "paid off" somehow, by exporting assets, either equities or money.

The discrepancies between the two curves in Fig. 5.24 on a quarter by quarter basis, shown in Fig. 5.25, are not surprising, since transfers of assets may not necessarily occur at the same time as the transfers of goods and services. However, it would be expected that on average the discrepancies would cancel each other. This is apparently not the case. Fig. 5.26 shows the *cumulative discrepancy*. It would be expected that such curve would be hovering around zero. Obviously it does not (although the net amount is not very high compared with cumulative value of the overall trade). There are multiple possible justifications for the result:

- **data collection may not be completely consistent**; data comes from very different sources, with possible inconsistencies;
- **changes in currency exchange rates** may lead to one transfer being recorded at one rate and the corresponding balancing transfer at a different rate;
- there are **invisible transactions** that are not properly recorded, in particular due to the activities of tourists, who may be purchasing goods and consuming services in foreign countries without accurate accounting.
- **illegal traffic**, particularly of drugs, may lead to considerable discrepancies in the flow of goods and money.

5.7 National Net Worth

The **USA National Net Worth** is the sum of the value of the **USA Domestic Assets** and all of the **USA Financial Assets**. The latter consist of 4 components, namely

1. the USA ownership of **ROW equities**, i.e. ownership by USA residents of ROW located assets;
2. the ROW ownership of **USA equities**, i.e. ownership by ROW residents of USA located assets;
3. USA resident **net** ownership of USA money;
4. USA resident **net** ownership of ROW money.

Fig. 5.27 shows the values of the two **equity** components and the net total. In the 50's the USA owned *less* foreign assets than foreigners owned USA assets. The situation changed in the 60's and since then the USA has consistently owned more foreign assets than vice versa. By 2010 the USA owned about 5 trillion $ more ROW equity assets than the ROW owned of USA equity assets.

Fig. 5.28 shows the values of the two **money** components and the net total. Up to the early 70's the USA owned a *positive* amount of both types of money assets and was therefore a *net creditor* relative to the ROW. The situation started to change in the early 70's and by the early 80's both variables were negative, but very small. Then in the mid 80's the USA ownership of ROW money assets started to grow, but the ownership by the ROW of the USA money assets grew even faster, with the result that since then the USA has become an increasingly *net debtor* relative to the ROW. By 2010 the USA owed the ROW a net amount of about 5 trillion dollars.

Fig 5.29 shows the net values of both the equity and money assets and the total net value of the USA financial assets. The overall net USA financial position has been positive until 2004. It has then become negative, particularly since the 2008 financial crisis.

The effect on the **Net Worth** is shown in Fig. 5.30. Fig 5.31 shows the rate of change of the Net Worth. Note that the **rate of change in Net Worth has always been positive, except for a single quarter in 2009**. In other words, notwithstanding the vagaries of the GDP and the increases in the US net *negative* money assets, the total net value of US owned assets has always been increasing (with the noted exception). However, the rate of increase has been decreasing from about 4.5% per year in 1950 to about 2% per year in recent years.

Fig 5.32 shows the average **Net Worth per person** and Fig 5.33 shows the rate of change in the Net Worth per person.

5.8 Net National Product

The **Net Domestic Product** is the difference between the **Gross Domestic Product** and the **Asset Loss**. However, not all of such product actually ends up in the hands of the resident population of the USA. Some of that product is exported. In compensation, some additional product is imported, but the excess of such imports may have to be paid with financial assets, either equity assets or money assets. The **Net National Product** is the total net added value actually *earned* by the resident population of the USA. It consists of the sum of the **Domestic Consumption** and of the **Change in Net Worth**, i.e. of the total actually consumed plus the net increase or decrease in net assets actually owned by USA residents. The change in National Net Worth can also be referred to as the **Total Net Investments**. The difference between the *Total Net Investments* and the *Net Investments* that we discussed in section 5.4 is the *increase in value* (in constant dollars) of the real assets owned by the USA.

Fig. 5.34 and 5.35 show the **USA Total Net Investments** and the **USA Net National Product.** Fig. 5.36 shows the USA Net National Product and the USA Net Worth. The two variables have been growing at the same rate, with the Net Worth being approximately 3 times the value of the Net National Product. Fig. 5.37 shows the **USA Net National Product per person**. Similarly to the other related variables, the Net National product per person has increased by more than a factor of three over the 60 year period. In Fig 5.38 we show the year to year changes in the USA Net National Product per person. As the trend line indicates, on average the trend has been pretty constant over the years, with the usual major oscillations. Fig. 5.39 shows the ratio of the change in Net Worth to the Net National Product. That ratio represents the *true yearly average percentage savings* of the USA resident population. Quite clearly there has been a decrease in that ratio since the 70's, from about 13% to about 7%.

In other words, over the last decades, the USA residents have consistently consumed more of their aggregate production and saved less.

5.9 Government Net Worth

In our model we have assumed that all *industrial* assets and liabilities are ultimately owned by the household sector. Therefore, the **Total Net Worth** is the sum of only two components, namely the **Government Net Worth** and the **Household Net Worth**, with the latter representing by far the largest component. In Fig. 5.40 we show both the value of the real assets owned by the government and the value (negative) of the government financial assets. In Fig. 5.41 we show the Government Net Worth. It is obvious the Government Net Worth has been going through a major oscillation. However, the figures are misleading. In Fig. 5.42 we show the *changes* in both the Government and the Household Net Worth. It is quite evident that the two changes are inversely correlated. Whenever there is a decrease or a smaller increase in the Government Net Worth, there is a corresponding larger increase in the Household Net Worth and vice versa. The reason is quite obvious. Typically, the Government Net Worth decreases when the government increases its debt by issuing more bonds. But these bonds are, by and large, purchased, directly or indirectly, by the household sector (except for those purchased by the ROW). Therefore they have no effect on the Total Net Worth. As shown in Fig. 5.31, the changes in total net worth have oscillated over the years, but they have been essentially *always positive*. In other words, the *total* net worth has been increasing monotonically over the whole period, independently of the major increases in the government debt.

One problem is that the "official" government assets consist essentially of some real estate and certain other assets. The fact that the federal government "owns" a large portion of the western states' land is not taken into account, since those lands are not "on the market". They are however valuable. Therefore the government assets are considerably undervalued.

Chapter 6

Productivity

In the previous chapters we have looked at the history of certain important economic variables, without much elaboration. We will now start to look at relations among some of those variables.

6.1 Business Employee Productivity

In the economic press there is often mention of *productivity*. What is meant is the average value of goods and services produced by an employee, measured either by the hour or the year of work. For various reasons this is normally restricted to *business employees*. This is not a significant restriction, since the industrial component of the GDP represents about 75% of the total. We will define **gross business employee productivity** as the ratio of business GDP in a given year to the average number of business employee in that year, i.e.

gross business employee productivity = business GDP/business employees

Fig. 6.1 shows the value of gross business employee productivity over the years. The graph shows that the productivity has increased by a factor of 2.5 over the 60 year period. However, the rate of improvement has been quite irregular, as shown in Fig. 6.2 that shows the yearly changes in productivity. The trend line indicates that for all practical purposes the rate of improvement has remained essentially constant. However, focusing on employee productivity, as defined above, is somewhat misleading, as we will show below.

6.2 Business Assets and Productivity

Even in the most elementary economics textbooks it will be noted that *production* is a function of two primary factors, namely *labor* and *capital*. To look at production only as a function of labor, as the standard definition of productivity used above would imply, is misleading. We obviously need to look at the other side of the production relationship, namely at the contribution of **business assets** to productivity.

Fig. 6.3 shows the labor productivity already shown in Fig 6.1 together with the value of business production assets available *on average* to each business employee. It is quite obvious that the two variables have increased together. A different way of looking to the same data is by looking at the relationship between **gross production/employee** and **assets/employee**. The two variables are shown in Fig. 6.4, together with the best linear fit. The best linear fit provides the relationship

Business GDP/Employee = 12.6 + 0.62*Business Assets/Employee

where the business GDP/employee and the assets/employee are expressed in thousands of dollars. The above relationship can be recast as follows:

Business GDP = 12.6*Number of Business Employees + 0.62*Business Assets

The above relationship says that the *incremental* production due to adding an employee (without adding additional assets) is about 12,600 $/year. On the other hand, the *incremental* production of adding a $ of assets (without adding any new employees) is about 0.62 $/year. We will call the first component the **direct labor productivity** and the second the **assets productivity**. Fig. 6.5 shows the *actual* and *estimated* values of the business GDP/employee, according to the above relationship. The error between the two is approximately 5.5%, which is not too bad. It should be remembered that the above relationship should only be considered valid only within the actual range of the variables that was observed. It would be inappropriate to actually extrapolate the relationship beyond that range.

In Fig. 6.6 we show the actual value of the business GDP together with the estimated value, using the last equation given above.

It is important to note that the above relationships should not be viewed as *short term* predictors of business productivity, but only as estimates of the *long term underlying labor and asset productivity*. At any given time the *actual current* value of the business GDP is affected by many other short term factors.

The estimated values are systematically underestimating the actual value of the business productivity around 1970. They overestimate it in the 80's. This indicates that there are other effects that should be taken into account. The most obvious one is that our analysis used data about the whole business sector as a single aggregate. It is not unreasonable to assume that for each *component activity* of the business sector there may be a relationship of the same structure as the one outlined above for the whole sector, i.e.

Component GDP = a*Number of Component Employees +
b*Component Assets

each component having a pair of component specific coefficients **a** and **b**. However, even as the component specific coefficients may remain constant, as the composition of the business sector changes over time, the resulting *overall* coefficients would certainly change. Unfortunately we do not have the detailed data necessary to perform a more in depth analysis.

Another factor to be taken into account is that the *effective utilization* of both *people* and *assets* may be affected by a variety of incidental factors which would somewhat distort the validity of the estimates. In particular, any significant reduction in employment would surely imply a lower utilization of assets, at least temporarily.

It is sometimes said that the increase in employee productivity is due to the development of a "better educated" labor force. It is possible to argue that some level of higher education may be necessary in order to properly utilize additional assets. However, the above analysis would lead to the basic conclusion that

the major factor determining the increase in employee productivity is the availability to employees of additional production assets.

6.4 Business Assets and Employment

For a different perspective let's look again at the trend in the value of business assets per employee as a function of time, as shown in Fig. 6.7, where we have added a line indicating the best linear fit. The equation for the best fit is

Estimated Assets/Employee = (37,500 + 1,450*(t-1950)) $

This constant rate of increase suggests that at any given time there is an "optimal" ratio of assets to employees and that that ratio is increasing approximately linearly with time. It is not unreasonable to take that linear fit as an estimate of the underlying long term trend. We can then estimate what the long term "sustainable" business employment is, given the actual value of the total business assets. In other words

Sustainable Number of Business Employees =
Business Assets/(Estimated Assets/Employee)

Fig. 6.8 shows both the *actual* business employment and the "*sustainable*" employment. The data would indicate that in the mid to late 90' there was "excessive" employment. The situation was corrected in the 2000-2008 period. This is consistent with the observation we made in Chapter 3 about the fact that starting in the year 2000 there was a significant reduction in the labor force.

6.4 Net Production and Compensation

In the previous sections we looked at the patterns of *gross* productivity, i.e. using the values of the Gross Domestic Product (GDP). We will now look at the situation with regard to *net* productivity, i.e. by taking into account the Net Domestic Product (NDP). The difference between the two is due to the loss of assets during the production cycle, i.e.

NDP = GDP – Asset Loss

The net employee productivity is obviously defined as

net business employee productivity = business NDP/business employees

Fig. 6.9 shows the history of the net business productivity over the years. Fig 6.10 shows the relationship to the assets per employee, similarly to what we have shown in Fig. 6.4 for the gross productivity.

The best linear fit is slightly different than the previous one, namely

business NDP/employee = 13.6 + 0.53*business assets/employee

This means that, when taking the NDP into consideration, the *direct* labor productivity is 13,600 $/year/employee and that the *assets* productivity is 0.53 $/year/asset $.

What does this relationship really mean? If an employer adds a new employee, it can expect an increase in net production worth 13,600 a year. If an employer adds 26,000 $ of assets, it can expect approximately the same increase in net production. Remember that all our $ values are expressed in constant 2010 $. Therefore we can assume that the employer can borrow the money to buy the assets at the "real" interest rate. Let's say about 5%. In addition, the employer must take into account the actual depreciation of the assets, which is on the average about 5%. So the total cost of adding the assets is about 10%/year or about 2,600 $/year. This is a less than $1/5^{th}$ of what it might have to pay a new employee, which would not be less than the 13,600 $ that the employee can claim that it is its net contribution to production (and actually would be much more, as we will see in a moment).

Quite obviously, the above argument cannot be pushed to the limit. As we mentioned a little earlier, there is an "optimal" balance between labor and capital and our employer cannot stray too much from it.

The NDP represents the *net added value* due to the business activities. The total contributions of labor and assets, according to the above estimated linear fit, are shown in Fig. 6.11, while Fig. 6.12 shows the respective *relative* contributions. Quite clearly the labor contribution has been decreasing drastically over the years.

Such added value *should* be redistributed among the factors contributing to its generation, i.e. "*labor*" (i.e. employees) and "*capital*" (i.e. the owners of the production assets). Unfortunately the current US political system has introduced a third factor into the equation, namely "*the citizens*", as represented by the *government*, through the latter collection of *taxes*. Naturally, government taxes will go eventually into delivering services to all the people, including both the wage earners and the owners of the production assets.

Part of the tax paid by business consists of their contribution to Social Security. This contribution is equivalent to a deferred payment to labor, so it should be added to the labor compensation. Under this assumption the repartition of the business net income (essentially equivalent to the NDP) is shown in Fig. 6.13 in absolute terms and in Fig 6.14 in relative terms.

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Quite clearly there is a major discrepancy between the valued added contributions and the actual compensations of the labor and assets production factors.

Fig. 6.15 shows the average rate paid to business employees. It shows that business employees nominal compensation has increased from about 32,000 $ in 1950 (in 2010 $) to somewhat over 70,000 $ by 2010.

The corresponding returns on assets are shown in Fig. 6.16. Returns have been increasing from about 5% in 1950 to about 10% by 2010, with most of the increase occurring in the last 10 years. The previous relationship for the business NDP/employee indicated that on the average a dollar of assets produced an added value of 0.53 $/year. In other words, labor has claimed as its compensation over 80% of the NDP that can be attributed to the assets themselves.

To make the point somewhat clearer, let's consider an analogy. Suppose that the only job that there is to be done is to move dirt from point A to point B. John Smith, using a shovel and a wheelbarrow gets to work. Working 8 hours a day, five days a week, 50 weeks a year, he manages to move in a year a certain amount of dirt. This work is worth 13,600 dollars/year and he is so compensated. Mary Johnson has a piece of earthmoving equipment worth 100,000 dollars. She gives it to John Smith who, working 8 hours a day, 5 days a week, 50 weeks a year, manages to do move much more dirt. After subtracting the cost of fuel and of maintenance for the equipment, the work done is worth 66,600 dollars, i.e. 53,000 dollars more than the previous case. The issue is, how much of the 53,000 dollars should go to John and how much to Mary? After all, John did not spend any more time working and actually his work was much easier to do, only having to move a few levers in the cab of the equipment. Why should he be paid anything more than before? In actual reality, John gets paid about 56,000 dollars, i.e. he gets about 43,000 dollars of the extra 53,000 that was produced, while Mary only gets about 10,000 dollars.

Is this fair? Somebody with an "extreme capitalistic" point of view might argue that labor compensation should only corresponds to the direct labor productivity and that all value attributable to the assets productivity should be translated into compensation for the owners of the assets. This is untenable on two grounds.

First, we should remember that the linear relationship we identified above between NDP and assets can only be considered valid within the historical range of the variables in question. Quite clearly, without labor, the productivity of assets would not match the value that might be extrapolated by setting the value of labor to zero. Therefore, we cannot reasonably claim that what we have called the "assets productivity" is truly due *exclusively* to the presence of the production assets, but is clearly due to the *combined presence* of both assets and labor. Second, if labor would be compensated the same, independently of the value of the production assets at its disposal, there would be no incentive for labor to accept the added responsibility of working with additional production assets, therefore eventually negating the value to the "capitalist" of providing such added assets to the employees.

It is therefore reasonable that the added value corresponding to the assets productivity be split between labor and capital. However, what is the "fair" ratio is undetermined. More realistically, there is no "fair" ratio. The ratio is determined by the relative economic and political strengths of the two groups, i.e. "labor" and "capital". If the ratio would be determined purely on the basis of economic supply and demand parameters, we would be led to conclude that in the US there is, *relatively speaking*, a *shortage of labor* and an *overabundance of capital*. There is at least circumstantial evidence that such is the case. First, as we have shown in Fig. 4.3, the percentage of employed people among people of working age has increased significantly over the years, from about 60% to about 70%. Second, there has been a considerable influx of immigrants, both legal and illegal, particularly from Mexico. These facts would appear to indicate the demand for additional labor.

In the last 20 years or so there has been an outflow of employment opportunities from the USA to the ROW. Although we have no actual data, it appears reasonable to presume that the joint productivity of *foreign assets* and *foreign labor* may be quite comparable to the one discussed above for *domestic assets* and *domestic labor*, at least for *some* business activities. It appears that foreign labor is willing to accept a repartition of compensation between labor and capital which is more favorable to capital than the one in the USA. This is a natural result of the fact that on average in the ROW there may an overabundance of labor and a scarcity of capital. Under these conditions it would appear natural that there would be an outflow of employment opportunities.

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# Chapter 7

# Government Financing

In this chapter we will analyze the effects of government financing. In order to properly compare their values over time some quantities may be expressed as a fraction either of the **Gross Domestic Product** (**GDP**) or of the **Net National Product** (**NNP**), as it would appear most appropriate.

## 7.1 The Government Budget Balance

A quantity that is often assumed to play an important role in the economy is the **Government Budget Balance**, which is the difference between the Government's income and the Government's expenses. For many reasons, expenses tend to exceed income more frequently than not. Therefore, most of the times the Government is in "deficit" (although it was briefly in "surplus" in the late 90's). Remember that in our definition, "Government" is the aggregate of the Federal, State and local governments. Fig. 7.1 shows the budget balance of the federal Government and the combined budget balance of the State and local governments. It is interesting to note that while the federal budget has been in deficit most of the times, the State and local governments were running a surplus up to the 70's. Fig. 7.2 shows the aggregate Government budget balance as a fraction of the GDP.

Quite clearly there has been a substantial change in the mid 70's, with the Government running very high deficits until the late 90's, when actually the overall budget balance became positive. As we said, the budget balance is the difference between income and expenses. It is interesting to look separately at the trends in the two variables as shown in Fig. 7.3.

The total Government income was about 25% of GDP in 1950; it then increased slowly to about 28% up to the 1980's where it stabilized until the middle 1990's when it increased again up to about 32% in the year 2000. The ratio then decreased to about 27% in 2004, when it started to increase to reach 30% in 2008, when it decreased again to about 26% in 2010.

Total Government expenses were also about 25% in the 50's, but then they grew steadily until they reached about 35% in the middle 80's, where they remained up to early 1990's. Expenses then decrease steadily until 2001 when they started to increase again, particularly after 2007, until the current level of about 39%.

At the beginning of the 90's we had an unusual combination of circumstances. The administration of George Bush senior decided on a tax increase (which was one of the
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reasons for Bush senior's reelection loss). This led to an increase in revenues to about 32% of GDP in the year 2000, *the highest that it had ever been since 1950*. At the same time a republican dominated congress cut expenses, bringing the expense ratio down to 30% of GDP in the year 2000, therefore producing a couple of years of budget surplus. The situation changed again after the George Bush junior's tax cut that brought the revenue down and a rebounding of the expenses, partly as the result of the events associated with 9/11.

7.1.1 The Social Security issue

The above picture is somewhat misleading. One significant item in the Government budget is **Social Security** that appears both as an income (from the tax paid by current employees) and an expense (as payments to retired people).

The official position is that Social Security is a form of "government pension". People are paying into the system when they are working and when they retire they will get an annuity from the government somewhat proportional to their contributions. The myth is that the money paid into the system is "invested" and later the invested capital, together with the returns on that investment, is used to pay back the annuities. This is not what happens. A bookkeeping entry is made into a "trust fund account", and the money paid into the system goes into the federal government coffers and it is used to pay all general federal government expenses. Some people have referred to the Social Security systems a "Ponzi scheme". This is not accurate. In a traditional Ponzi scheme, early investors are paid interests on their capital by using the capital of new investors, a process that cannot be sustained for long. This is not how the Social security system works.

Up to the 1800's, when a person became unable to earn its living because of old age, that person would be taken care by one or more of its children, or in absence of children, by some other relative. It was common in those times for "extended families" to live together, with multiple generations sharing common facilities. With the advent of the industrial revolution, people started moving away from their parents' household, therefore creating a problem for the older people when they became unable to support themselves.

In 1889 Bismarck, the Chancellor of Germany, first promoted the concept of the state taking the financial responsibility for the old people, by creating the first "old age state pension system". Other nations followed the example later. The US Social Security system was created in the 1930's. In all such systems, it is still the young generation that pays for the upkeep of the older generations, like it was done since time immemorial. The difference is only that the state (the "government" in the US) acts as intermediary, by collecting the money, usually in the form of payroll taxes, and then redistributing it among the older people.

This is a straightforward **redistribution of income**, in which the Government only plays a money shifting role.

Until 1966, the Social Security accounting was kept separate from the general federal budget. During the Johnson administration it was "discovered" that the Social Security budget was in surplus, because the payroll tax rates were such that they brought in more money that was paid out to retired people. It was then decided to merge the accounting with the general budget, therefore reducing the "official" federal budget deficit. Of course, there was nothing different in practice: the "real" government budget balance remained the same, only the formal accounting was changed.

The Social Security budget balance is shown in Fig. 7.4. The budget has been consistently in surplus until the very latest years, when the skyrocketing Medicare costs have resulted in a considerable deficit. The relative role of the Social Security budget has gone from about 1% of GDP (averaging income and expenses) to about 7%. The change is quite significant.

Fig. 7.5 shows the Government income and expenses, after subtraction the Social Security components. It shows a much more stable income stream, averaging approximately 24% of GDP with a variation of about 2%. Expenses on the other hand have grown from about 24% of GDP to about 28%. It shows also that the surpluses around the year 2000 were largely due to the surplus in the Social Security budget.

7.3 The Government Debt

Since the Government budget is almost always negative, the Government accrues an increasing debt. Fig. 7.6 shows the history of the Government debt relative to the GDP. It should be noted that *in constant dollars* the Government accumulated debt is also affected by the rate of inflation. The higher the rate of inflation, the more the value of the *past* Government debt will be reduced *in constant dollars*. It is conceivable that after a year in which the Government has had a deficit, the total debt in constant dollars may actually be *reduced*, if the inflation rate has been high enough!

Fig 7.7 shows what is the *cost* to the government of financing the government debt, again relative to the GDP. From 1950 to 1980 the ratio was approximately constant, around 1.5% of GDP. Then there was a sharp increase, with the cost of financing reaching 3.5% in the 1990's. Then the cost decreased again back to approximately 1.5%. It should be noted that, notwithstanding the increase in debt of the last few years, the cost of financing such debt is again close to what it was in the 1950 to 1980 period.

In recent months (summer 2011) there has been in the news a repeated, deafening bombardment of assertions about the so called "National Debt". Almost everything that is being said is idiotic. We will attempt to clarify what we believe to be the reality of the situation.

First of all let's clarify some terminology. What is the meaning of the word "Nation"? Depending on the context, it usually means a **group of people** (the **citizens** of the nation) or a **geographical area** (the **territory** of the nation). In the context of the US economic data a lot of the information is available not on the basis of the *citizens* of the US, but instead on the basis of the **residents of the territory of the US**, sometimes including the US overseas military. The difference does not really matters for our analysis, so we will refer generically to the **US people.**

There is sometimes confusion in the use of the word "government". As we have outlined earlier, in the US there is one **Federal Government**, there are 50 **State Governments** and some thousands of **Local Governments**. From the political point of view it may be important which level of government does what, but from the overall economic point of view it does not matter which government level raises which taxes and which government level provide which services. Therefore when we will refer to "**Government**", we will mean the sum total of the three levels of government just mentioned, unless we explicitly distinguish among them.

In the US the Government is perceived as an agency of the people. It performs those functions that the people have chosen to assign to it. The Government **is not the nation**.

The US Nation is the ensemble of the US Government and of the US people.

The "government debt" is not the "national debt". We will clarify this issue later on.

In order to make our point of view easier to understand we will first consider some hypothetical situations, before getting to the actual situation of interest.

7.3.1 The US as a closed economy

Consider for the moment a hypothetical US Nation that is completely closed from the economical point of view. This means that there are no imports and no exports. Also it means that the US people cannot own any foreign property and that no foreigner can own any US property.

This US nation has US located **real assets** valued at 40 trillion dollars. By "real assets" we mean land, buildings, machinery, etc. Of these assets, 10 trillion dollars are owned by the US Government and 30 trillion by the US people directly. The US GDP is 15 trillion dollars, which translates into a people's income of approximately 14 trillion dollars (the difference is mainly due to the cost of replacing consumed assets). The Government raises 4 trillion dollars in taxes and we assume for the time being that it has a perfectly balanced budget, spending exactly 4 trillion dollars to provide its services. The people are left with 10 trillion dollars of **disposable income**, which they can use for consumption and investment. We assume that the Government has no debt. The situation is summarized below:

People's real assets	30.0
Government real assets	10.0
Nation net worth	**40.0**
Total GDP	15.0
Gross income	14.0
Taxes	-4.0
Disposable income	**10.0**
Government tax income	4.0
Government expenses	-4.0
Government budget balance	**0.0**

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Everybody is happy. Well, not everybody. There always be some people that would prefer to pay less taxes (and get fewer government services) while other people may prefer to get more government services (and of course pay more taxes, particularly if they can get somebody else to pay them). But from the strictly *financial* point of view, everybody is happy.

Now let's modify the situation a little. Let's suppose that in times past the Government has ran considerable deficits and has therefore accumulated a debt of about 12 trillion dollars. This means that the Government has had to issue 12 trillion dollars of Treasury bonds. Since the economy is closed, the US people are the ones that own all of the 12 trillion dollars in bonds. This means that the **net worth of the Government** is *minus* 2 trillion dollars (10 minus 12) while the **net worth of the people** is 42 trillion dollars (30 plus 12). The **net worth of the *nation*** is 40 trillion dollars, just as before. It should be noted that while the *government debt* is 12 trillion dollars, the people have an *aggregate credit* of 12 trillion dollars, therefore the *national debt*, i.e. the combined debt of the government and of the people, is exactly zero. To service its debt we assume that the Government must pay every year 5% interest, i.e. a total of 0.6 trillion dollars. We assume that the Government wants to maintain a balanced budget, therefore it has to raise an additional 0.6 trillion dollars in taxes. But now the people's income is 14.6 trillion dollars, i.e. the previously 14 trillion dollars derived from the economic activities, plus the 0.6 trillion that it receives in interest income. Therefore, after paying the new total of 4.6 trillion dollars, the people are left with exactly the same 10 trillion dollars they had before. In summary:

| | |
|---|---|
| People's real assets | 30.0 |
| People's money assets | 12.0 |
| Government real assets | 10.0 |
| Government money assets | -12.0 |
| **Nation net worth** | **40.0** |
| | |
| Total GDP | 15.0 |
| Gross income | 14.6 |
| Taxes | -4.6 |
| **Disposable income** | **10.0** |
| | |
| Government tax income | 4.6 |
| Government expenses | -4.6 |
| **Government budget balance** | **0.0** |

Therefore there is no difference from the previous case. It should be noted that the above arithmetic works for **any** amount of Government debt. If the Government debt were to be 100 trillion dollars, it would not make any difference. Therefore we can conclude that in a closed economy

**the Government debt has no <u>direct</u> effect**
**on the <u>overall</u> underlying economy**

It is often said that by allowing the Government to be in debt we are "burdening" future generations with the responsibility for such debt. I cannot understand how such idiocy can be asserted. In a closed economy future generations will inherit both the *government debt*
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and the Treasury bonds which represent the *corresponding credit*. So the effect of the debt will be null for them as it is for the current generation.

Now let's modify the situation a little further. Let's consider the case that the Government is not so keen on a balanced budget, but it may consider a budget deficit. When talking about government budget deficits, it is common to say that the problem is that the government spends more money than it gets as income and it is therefore "financially irresponsible". An analogy is often made between the US Government and a "private family". The analogy is totally wrong. A typical family has a given income, derived in most cases by the work of one or two members of the family. Out of that income the family will pay some taxes. What is left is the "disposable income". A responsible family will develop a "budget" that will allow for some fraction of that disposable income to be used to pay for its consumption needs and the remainder to be saved in order to build a nest egg. A family that spends more than it earns will build a debt that eventually will have to be paid off. It cannot continue to spend more than its disposable income on a permanent basis.

Some people claim that the Government should behave like a responsible family. I believe that this is totally misleading. **The Government does not have a predetermined income**. For the Government the budget procedure is exactly the opposite of that of a family. **It is the *spending* that it is decided first**. The level of spending is determined by the desires of various politically influential groups to have the government provide certain services. Some groups may want a strong defense establishment, some other groups want to guarantee a safety net for indigent or disadvantaged people, some other groups want to establish and enforce strict environmental standards, and so on. Many of these desires are popular with the electorate. Therefore eventually laws are passed establishing certain government services which imply certain expenditures. When the time comes to finance the total of such expenses a new set of issues arises. Everybody wants to pay the least possible taxes, however it is perfectly happy to see *other people* pay more taxes. Voting for additional taxes is not politically desirable for an elected representative; therefore it is easier to allow for deficit financing. In other words, the issue is that the spending is predetermined and the choice which is left is on how to *finance* such expenditures. There are basically three ways to do it: by taxes, by issuing bonds or by printing money. In the case of the US the latter option is considered officially unacceptable under normal conditions. So the choice is between raising taxes and issuing bonds. Let's see what the difference really is between the latter two.

So let's go back to our example. Let's assume that the Government decides that it does not want to raise taxes to 4.6 trillion dollars, but that instead it decides to *reduce* taxes to 3.6 trillion dollars and to issue bonds for the needed additional 1 trillion dollars. We have the following situation: the people have an income (including interest on the debt) of 14.6 trillion dollars. After taxes they now have 11 trillion dollars left. However, they *must* buy 1 trillion dollars' worth of bonds. After paying for that, they are left again with 10 trillion dollars to be used for both investing and consumption. So we are back to the same situation. When the people pay taxes they get (at least in principle) a "thank you" note from the government. When people buy bonds, they a get a piece of paper (the bond) saying that the government owes them some money. But the money they send the government is the same.

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People’s real assets	30.0
People’s money assets	12.0
Government real assets	10.0
Government money assets	-12.0
Nation net worth	**40.0**
Total GDP	15.0
Gross income	14.6
Taxes	-3.6
Net income	11.0
Purchase of bonds	-1.0
Disposable income	**10.0**
Government tax income	3.6
Government expenses	-4.6
Government budget balance	**-1.0**
Government bond income	1.0
Government cash balance	**0.0**

There is however one difference, namely the following year the government will have a debt of 13 trillion dollars. This will mean that the people income will be increased by another 50 billion dollars of interest to 14.65 trillion, but that they will have to send the government, by taxes or bond purchases 4.65 trillion dollars, ending back up in the same place.

Lets’ consider now the opposite situation, i.e. the Government decides to pay off some of the debt by raising taxes, i.e. producing a budget *surplus*. Let’s say it raises taxes to 5.6 trillion dollars in order to buy back 1 trillion dollars’ worth of bonds. After paying taxes, the people will now be left with only 9 trillion dollars (14.6 minus 5.6). However they will get a 1 trillion dollar cash back payment from the government, paying off the bonds.

People’s real assets	30.0
People’s money assets	12.0
Government real assets	10.0
Government money assets	-12.0
Nation net worth	**40.0**
Total GDP	15.0
Gross income	14.6
Taxes	-5.6
Net income	9.0
Income from bonds	1.0
Disposable income	**10.0**
Government tax income	5.6
Government expenses	-4.6
Government budget balance	**1.0**
Government bond payment	-1.0
Government cash balance	**0.0**

Therefore their disposable income will be again 10 trillion dollars. The difference is that the following year the government debt will be 11 trillion dollars. This will mean that the people income will be lowered to 14.55 trillion dollars and that they will have to pay only 4.55 trillion dollars in taxes or bonds, again leaving them the same 10 trillion dollars.

But wait a moment, you say; in this way the US people are "consuming" part of what was their "financial asset", namely the money that they had "invested" in the treasury bonds. Surely that must be bad! The issue here is that what may be true for an individual person it may not be true for the whole of the people, as a community. Treasury bonds may be seen as an investment by an individual, since they provide (in our example) a steady 5% interest, like many other investments might. Somebody must be paying additional taxes so that the investor may get its money, but that somebody is probably somebody else, so the individual owner of the Treasury bonds does not care. However, when we look at the situation from the point of view of the entire community, the situation is quite different. It is true that the whole of the people receive a certain amount of interest, but that interest is paid by the entire community through extra taxes. To the community as a whole, the existence of Treasury bonds has no net financial effect. Therefore:

**from the point of view of the people as a whole,
Treasury bonds are NOT an investment**

A different way to put it is that Treasury bonds represent a loan that the people make to themselves, essentially moving money from one pocket to another.

In summary in a closed economy:

**the Government debt and budget balance (either deficit or surplus)
have no <u>direct</u> effect on the <u>overall</u> underlying economy**

In the above bold statements (both in content and in text) we have underlined the word "overall". That is where the rub is. While in any circumstance we looked at there is no change *in the aggregate*, it does not mean that there are not different implications for different people. In all cases there will be people who gain some and people who lose some. But this is inherent in *any* government activity. It will be very unusual to find anyone who gets back from the government in services exactly the value that it pays in taxes. In almost all cases some people will pay more in taxes than the value of the services they receive from government while for some people the opposite will be true. Most commonly, the higher income people will pay more taxes and receive fewer services, while the low income people will pay less in taxes and receive more in services.

In the previous statements we have also underlined the word "direct". The reason is that while our assertions are correct at the first level of analysis, there is an issue of possibly *indirect* effects that might be of significance. We will look at some of these issues after we have looked more carefully to the actual case for the US economy.

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## 7.3.2 The actual US economy

Let's now look at the actual situation for the US. In the real world the US coexists with many other countries. For our discussion, from the pure economic point of view, it is of no interest which specific country is involved in which specific transaction. We may look at all of the foreign countries as a single entity, the "**rest-of-the-world**". We will refer to such entity as the **ROW**.

The US imports goods and services from the ROW and exports goods and services to the ROW. In addition, some US people own some assets physically located in the ROW' territory and ROW people own some assets physically located in the US territory. Finally, the US people may own bonds issued by ROW authorities, while ROW people may own bonds issued by US authorities. Unfortunately, most data regarding international transactions are only imprecisely known. Furthermore, the evaluation of assets actual values in US dollars is subject to changes in the exchange rates of the various currencies. The number that we are using below are believed to be a fairly good estimate, but should not be viewed as absolutely exact. However, for the purpose of our argument, absolute accuracy is not required.

The first thing is that we have to clarify what is the **US Nation Net Worth**. We start by adding in the value of all real assets situated in the US territory. However some of those assets are owned by foreigners. On the other hand, the US government and the US people own some of the ROW real assets. So we have to subtract the value of the assets owned by foreigners and add the value of the foreign real assets owned by the US nation. This defines the overall net value of the real assets owned by the US nation.

Now we have to look at *money assets* owned or owed by the US government and the US people. Such money assets come in the form of bonds, loans and other money financial instruments. For simplicity we will refer to them all as "bonds". The ROW may own some of the US issued bonds, but also the US may own some of bonds issued by the ROW. The numbers are as follows (in trillions of US dollars), at the end of 2009:

| | US Government | US People | US Nation |
|---|---|---|---|
| US located real assets | 9.0 | 29.8 | 38.8 |
| US assets owned by the ROW | 0.0 | -14.3 | -14.3 |
| ROW assets owned by US people | 0.0 | 19.0 | 19.0 |
| **Real assets** | **9.0** | **34.5** | **43.5** |
| US bonds net position | -10.4 | 2.2 | -8.2 |
| ROW bonds net position | 0.2 | 2.8 | 3.0 |
| **Net worth** | **-1.2** | **39.5** | **38.3** |

In the above table we have added under the heading of "US people" both the assets of the US households and of the US business sector. Note that the Government has a debt higher than the value of its assets, i.e. the US Government has a **negative net worth,** or, to use a recently popular expression, is "under water". About 8.2 trillion dollars of the US debt is
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held by the ROW. However, the US Government and the US people own together 3.0 trillion dollars of ROW debt. Totaling up the assets and money positions we have:

	US Government	US People	US Nation
Total assets	9.0	34.5	43.5
Net bonds position	-10.2	5.0	-5.2
Net worth	**-1.2**	**39.5**	**38.3**

The US nation owns real assets for a total value of 43.5 trillion dollars. However, against such assets it has a net debt of 5.2 trillion dollars. The net worth of the US Nation is 38.3 trillion dollars.

The net debt of the US nation toward the ROW is the true "national debt"

It should be noted that the 5.2 trillion dollars that the US owes to the ROW does not consist all of government debt. There are also US *private bonds* in the total amount of about 2.9 trillion dollars, some of which are held by US private owners and some of which are held by the ROW. The combined value of ROW owned government and commercial nods is however 5.2 trillion dollars.

We need here to open an important parenthesis. Of the total US Government debt, 8.9 trillion dollars are due to the federal Government, while 1.5 trillion dollars are due to the State and Local Governments. How come that the debt numbers which are usually mentioned for the Federal Government are in the range of 13 trillion? The reason is that the total value of bonds *issued* is indeed in the neighborhood of 13 trillion. However, about 4 trillion of those bonds are actually owned by the US Federal government itself (mostly by the US Federal Reserve)! In other words, about 4 trillion dollars appear twice in the federal government assets and liabilities budget: once as a "debt" (because of the issuance of the Treasury bonds) and once more as a "credit", because the Treasury bonds are held by the Government itself. The **net** Federal Government debt is, as stated above about 8.9 trillion dollars.

To service the national debt of 5.2 trillion dollars, the US nation pays to the ROW about 150 billion dollars a year, corresponding to an average interest of 2.9%. The 150 billion dollars represent 1.2% of the US Net National Product of 13.4 trillion dollars,

The portion of the government debt that is held by the US people is in the same situation as what we have described in the section on the hypothetical "closed economy US", i.e. it is essentially irrelevant. What really counts, is the net total that the US nation owes to the ROW and that is, as stated above, about 5.2 trillion dollars.

How bad is that? To make it more comprehensible we can draw an analogy with an individual home owner. The situation of the US nation is about equivalent to a person owning a home valued at about 435,000 dollars and having a mortgage of about 52,000 dollars. To service the debt the person is paying 1.500 dollars of interest a year, out of a gross annual income of about 134,000 dollars. I do not think that there would be a single person that would consider that home owner as “financially overexposed”.

7.4 Government Financing and the NNP

One interesting issue is however, does the Government choice of financing strategy affect the “real” economy? Fig. 7.8 shows the relation between the government *budget balance* (relative to GDP) and the change in the Net National Product. The data shows a minor positive correlation, indicating that a high deficit could be correlated with a reduction in the NNP. In the last two years, 2008 and 2009, the Government has had an exceptional high deficit, due in large part to the financial bailout costs and the cost of the economic stimulus. If we take out the data from the last 2 years, 2008 and 2009, the data in Fig. 7.8a shows a perfectly flat trendline, indicating lack of correlation. The “traditional” view is that a Government budget deficit would contribute to “pump up” the economy, possibly leading to an increase in NNP. The reason why this is apparently the case may be very simple. The “traditional view” was based on the assumption that the deficit was financed by “printing money”, therefore introducing more money into the economy. In the US, the deficits are generally financed by selling treasury bonds. This means that no additional money is actually introduced in the economy, as we have discussed earlier.

Fig 7.9 shows the relation between the government *debt* and the NNP. Again it shows no significant correlation. I believe that we can safely conclude that

the Government budget balance and the Government debt have no practical effect on the Net National Product

In other words, the ***total* net aggregate** of the goods and services produced by the US economy is unaffected by the Government financing strategy.

There is however a real problem and we are going to discuss it in the next section.

7.5 The real problem

As we have already mentioned, the **US Net National Product** characterizes the total net production of goods and services which are available to the US nation in any given year. A slightly different, but interesting statistic is the **US Net National *structural* Product (NNSP)** which consists of the NNP *minus* the contribution due to the net income or expense due to the net interests paid to (or received from) the ROW. In other words, the NNSP measures the actual net production of the US economy, before taking into account the income or payments due to past positive or negative money financial position.

The **US Net Worth**, as discussed earlier, measures the net aggregate real and financial assets of the US. However, from the point of view of overall national production of goods and services what it really counts is the **US Net Real Assets**, i.e. the US Net Worth *minus* the net contribution of the financial money assets.

Fig. 7.10 shows the history of the US NNSP and the US Net Real Assets, with the two variables measured on the two different vertical axis. Quite clearly the two variables are highly correlated. Fig. 7.11 shows the value of the NNSP relative to the net real assets, showing that the NNSP has been equal to about one third of the value of the net real assets (more precisely, 31.8% on average).

In other words, the US NNSP in any given year is primarily determined by the US Net Real Assets and for every dollar of real assets we get approximately 32 cents of NNSP or an overall net return on investment of 32%. Of course there are minor deviations from this general rule. These deviations are due to the actual effectiveness with which the real assets of the US nation are utilized. This, in turn, is largely due to the level of employment. In particular, during the 2008 recession, while the value of the Net Real Assets increased, the sharp drop in employment led to a considerable drop in the actual utilization of the assets and therefore to a considerable drop in NNSP. However, the overall trend is quite obvious. The relationship entails that future increases in the US NNSP will be largely determined by the increase in US Net Real Assets, i.e. by the **net real investments** by the US nation.

Fig. 7.12 shows the history of US net real investments in the last 60 years. It shows that the US has achieved positive net investments every year, even in the presence of the deep recession of 2008. However, Fig 7.13 gives us a different view of the situation. It shows the net investments as a percentage of the NNSP. The **rate of investments** has been decreasing constantly, from about 13% in the 1950's to about 8.5% in 2000's. This means that the net worth of the US has been increasing more slowly than in the past and that therefore the net national product has been increasing more slowly than in the past.

Let's now look at the US Government activities. In Fig. 7.14 we show the relation between the Total US Government Expenses and the US Total Consumption, both relative to the NNP. It shows that as the Government expenses grow as a fraction of NNP, the overall fraction of NNP devoted to consumption also grows and therefore the fraction of NNP which is invested obviously decreases. This is shown more clearly in Fig. 7.15 which shows the same information, but now plotting the net investment rate vs. the total Government Expenses. For every 1% of increase in the ratio between the total government expenses and the net national product we have, on average, a decrease in of 0.5% in the rate of net investment. This is not good.

There is nothing fundamentally wrong with increasing the ratio of government expenses to the net national product. It is true that this is anathema to many people, but this is a *political* choice that the US people are free to make, if they so choose. The problem is that if the increase in the government role means an *increase* in the government portion of the total consumption, there must be a corresponding *decrease* in private consumption, if we want to maintain a given rate of investment. Clearly this has not been happening.

To clarify the issue we need to look in more detail to the government finances. In Fig. 7.16 we show the relationship between the Total Taxes paid to the Government and the Government expenses, always relative to the NNP. It shows that on average total taxes have been growing much more slowly than government expenses. On average, in the last 60 years US taxes have grown by about 0.25% for every 1% increase in total expenses. This has led to increasing deficits, as we have shown in Fig.7.3.

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In the first section of this paper we have argued that it did not really matter *in a closed economy*, if the government chooses to finance expense with taxes or debt, since in both cases the same amount of money is taken out of the pockets of the people. However, **this is not true if the US can sell bonds to the ROW**. In this case the ROW is contributing part of the money, not the US people. This is exactly was has happened.

There is nothing wrong in borrowing money. Actually borrowing money *to increase investments*, it is a very good thing to do. The nation would pay about 3% per year in interests, but the net national product would increase by 32%, with a net gain of 29%! The problem is that the US has used the borrowed money **to increase its current consumption** (therefore increasing its current standard of living), at the expense of **reducing its rate of investment** (therefore reducing the future potential growth in the standard of living).

To summarize the situation it appears that the following steps have taken place:

- the people have chosen to increase the services provided by the government;
- the increase in government services has led to an increase in government expenses;
- the increase in taxes paid to the government has not kept up with the increase in expenses;
- this has led to government deficits, financed by treasury bonds;
- a substantial portion of these bonds have been sold to the rest of the world;
- leaving the corresponding amount of money in the pockets of the US people;
- the US people have used that money for additional consumption;
- therefore reducing the rate of net investments in new assets;
- therefore reducing the rate of increase in net worth;
- therefore reducing the future increase in net national product.

In summary:

the US nation has been consuming too much and investing too little

and this is not good.

7.5 The Government Budget Balance and Inflation

Another issue that is often raised is about the relation between the Government budget balance and inflation. Fig. 7.17 shows the relevant data. Interestingly enough the weak relationship shown in Fig. 7.17 is in the opposite direction from the "traditional expectation", namely a higher deficit would appear to lead to *lower*, rather than *higher* inflation! How can such "perverse" relationship come to pass? The first answer is that the data is so much scattered that it does not make much sense to see a real pattern. A more general explanation is that the values of economic variables are determined by a large number of decisions made by many individuals and organizations. They are not determined by "blind" physical phenomena. This means that people adjust their behavior to changing circumstances. In other words the economic system has considerable "feedback" built in, which tends to create more complex relationships. I believe that it is reasonable to conclude that

the Government budget balance has no practical effect on the inflation rate

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# To be continued
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Appendix A

The Value of Money

The "value of money" is the *relative purchasing power* of the unit of money, i.e. the ability of the specified unit of money to purchase goods and services at a given time t_i *relative* to its ability to do so at some other time t_j. If there is a single commodity, quite clearly the value of money at t_i relative to its value at time t_j is determined straightforwardly by the ratio of the prices at the two different times. However, as soon as we have to deal with more than one commodity, the problem arises of how to "combine" the various prices into an overall measure. This implies obviously that the "value of money" can only be stated *relative to a specific set of commodities*. There has been considerable theoretical work on the issue, but the practical implementation of many ideas appears to have been lagging. W. E. Diewert and A. O. Nakamura (1993) provide a comprehensive review of the issue and an extensive bibliography.

I. Overview of the Current Situation

We will not even remotely attempt to review the current state of the theory and practice, but we will focus on a few specific issues. In all that follows we assume that for any given time interval t_i there is an *average price*, p_{im} for each commodity, *m,* and that the price is *predetermined*. In other words, we will not investigate in any way the factors that may affect the price of any commodity. We will only be concerned with the effect that the varying prices have on the *purchasing power* of the *money unit* at any given time. We assume that all transactions for each commodity during a given time interval occur at the average price for that interval. We will denote by q_{im} the quantity of each commodity purchased at time t_i.
A methodology that has received considerable attention from both the theoretical and practical point of view is the one based on *Fisher "ideal" price indexes*. Diewert (1987) provides a complete review from both the historical and theoretical points of view. The Fisher price index for time t_i relative to time t_j, $F(i,j)$ is defined as

(1) $$F(i,j) = \sqrt{\left(L(i,j) \times P(i,j)\right)}$$

where

(2) $$L(i,j) = \frac{\sum_m p_{im} q_{jm}}{\sum_m p_{jm} q_{jm}}$$

is the *Laspeyres* price index and

(3) $$P(i,j) = \frac{\sum_m p_{im} q_{im}}{\sum_m p_{jm} q_{im}}$$

is the *Paasche* price index. There has been considerable attention paid in the literature to the fact that the Fisher price index satisfies a number of a priori "tests" that have been put forth by a number of people, supposedly characterizing what a "good" index should do. Unfortunately, the Fisher price index fails to satisfy a number of other tests, both theoretical and pragmatical. In general the Fisher index does *not* satisfy the relation

(4) $$F(i,j) = F(i,k)F(k,j)$$

for arbitrary *i*, *j* and *k*. In other words it does not have the *transitivity property* (that in some economics literature has been sometimes referred to as the "circularity property"). This could be "fixed" by selecting a specific time interval, t_0 as the *reference time interval* and define the *relative* Fisher price index, $F_r(i,j)$ as

(5) $$F_r(i,j) = F(i,0)/F(j,0)\,.$$

However, because of lack of the transitivity property, the results would actually depend on what time interval would be selected as reference, making the whole process essentially meaningless. In addition, the Fisher price index has a conceptual difficulty in handling "new" commodities or "obsolete" commodities. If a commodity appears in the set of commodities for some time interval, t_i, but not for another time interval, t_j, it is not possible to compute the value of the Fisher index for those times since there is no price associated with the commodity at the time it is absent. The Fisher index is also inadequate in the way it handles the issue of *commodity substitution* associated with the drastic change in the relative prices of some commodities. An example will clarify the situation. Let there be only two commodities and let the prices and quantities for two time intervals t_1 and t_2 be

(6) $$p_{11} = 1; \quad p_{12} = 1; \quad q_{11} = 1; \quad q_{12} = 1;$$

(7) $$p_{21} = 1; \quad p_{22} = 10000; \quad q_{21} = 2; \quad q_{22} = 0;$$

The corresponding value of the Fisher price index is

(8) $$F(2,1) = \sqrt{\frac{10001 \times 2}{2 \times 2}} \cong 70.7$$

indicating that the "average price level" for t_2 relative to t_1 is seventy times higher, a result that does not appear acceptable (although what the "right" result "ought" to be is by no means obvious). The problems of the "missing commodity" and of drastic changes in prices are obviously more likely to occur the further the time intervals are separated in time, therefore making the choice of a fixed reference time interval less palatable in the context of a long time series.

In order to deal with this problem the notion of the Fisher *chain* price indexes has been introduced. There are two assumptions on which the idea is based, namely:

1. whenever a "new" commodity appears, at its first appearance it normally has a low quantity value associated with it and therefore it represents a small portion of the overall total purchasing value of all commodities (a symmetric argument exists for an "obsolete" commodity);
2. price changes occur gradually, so that the difference in relative commodity prices is never very large for time intervals closely spaced in real time.

We will refer to the Fisher price index defined in (1) as the Fisher standard price index. In the context of a series of time intervals, t_i, with i=1,2,….,N, we define the Fisher chain price index,
relative to a pre-designated time interval t_j, $F_c(i, j)$, recursively as follows:

(9) $$F_c(j, j) = 1$$

(10) $$F_c(j+k, j) = F(j+k, j+k-1)F_c(j+k-1, j)$$

(11) $$F_c(j-k, j) = F(j-k, j-k+1)F_c(j-k+1, j)$$

for $k \geq 1$.This implies that the Fisher standard price index needs only to be evaluated for consecutive time periods. If there is a "missing" commodity, we can deal with the problem by limiting the evaluation of the base index only to those commodities that are in common. In view of assumption 1, this will produce a negligible distortion. The Fisher chain index satisfies the transitivity property, i.e.

(12) $$F_c(i,k) = F_c(i, j)F_c\left(j,k\right)$$

How good is the new procedure at dealing with rapidly changing prices? Consider again the previous numerical example and assume that there are three intervening time periods, with the overall picture being as follows:

(13) $p_{11} = 1;\quad p_{12} = 1;\quad q_{11}=1;\quad q_{12}=1;$

(14) $p_{21} = 1;\quad p_{22} = 10;\quad q_{21}=1.5;\quad q_{22}=0.05;$

(15) $p_{31} = 1;\quad p_{32} = 100;\quad q_{31}=1.8;\quad q_{32}=0.002;$

(16) $p_{41} = 1;\quad p_{42} = 1000;\quad q_{41}=1.9;\quad q_{42}=0.0001;$

(17) $p_{51} = 1;\quad p_{52} = 10000;\quad q_{51}=2;\quad q_{52}=0;$

We have

(18) $$F(2,1) \cong 2.66;\quad F(3,2) \cong 1.89;\quad F(4,3) \cong 1.41;\quad F(5,4) \cong 1.20;$$

(19) $$F_c(5,1) \cong 8.51;\qquad F(5,1) \cong 70.7$$

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showing that the new procedure certainly helps in reducing the value of the price index for the extreme time points to a more reasonable value. However, the change appears to be one of degree, since the resulting value of $F_c(5,1)$ still appears to be "abnormally high".

There is, however, a major conceptual problem with the Fisher chain price indexes. Assume that at times t_i and t_j the prices for all the commodities are identical. If t_i and t_j are consecutive, (i.e. $i = j + 1$), we have that

(20) $$F_c(i, j) = 1$$

as it should be. However, if there is exactly one intermediate point, (i.e. $i = j + 2$), equation (20) will *not* hold in general *unless* also all of the quantities are the same for the two time intervals. If there are two or more intermediate points, (i.e. $i = j + k;\ k \geq 3$), then equation (20) will *not* hold in general even if the quantities are also the same. For example, let's assume that in the previous example the values for both prices and quantities at time t_5 are the same as for time t_1 . The value of $F_c(5,1)$ will be 0.31, rather than 1!

More generally, $F_c(i, j)$ for $i > j + 1$, will depend on the value of any of the intermediate time intervals between t_i and t_j. This means, among other things, that if we keep t_i and t_j fixed and change the number of time intervals between them for which we accumulate data (e.g. we change from yearly to quarterly data), the value of $F_c(i, j)$ will in general change. Such anomalies would appear to be unacceptable.

We have discussed at some length the Fisher price indexes for one primary reason. The Bureau of Economic Analysis (BEA) of the US Department of Commerce has the responsibility to evaluate the "real" value of the GDP and other related quantities on behalf of the US Government. It has chosen to use the Fisher chain price index methodology as its primary methodology (Landefeld (1997)), although in certain contexts it also occasionally refers to the Fisher standard price indexes, relative to selected years, for short time spans around the reference year. We believe that there are better choices and the major intent of this paper is to demonstrate one such alternative.

II. The Basic Framework

The basic idea of the approach that we will discuss was put forth in 1924 by A. A. Konüs (1924). Although the methodology proposed by Konüs has been extensively discussed in the literature, it appears not to have been applied in practice, for reasons that we may be able to clarify later on.

We will consider the problem from the point of view of a *single individual purchaser* of goods and services. We assume that there are M commodities, with prices p_{im} at time t_i and for commodity *m*. We will denote in general a vector of prices by the symbol $\mathbf{p}$ and the value of the vector at time t_i by $\mathbf{p_i}$. We assume that at each time t_i the individual has a non-zero *budget*, b_i which the individual *will* spend on the purchase of a selection of quantities of the different available commodities. We will denote by $\mathbf{q}$ a general vector of commodity quantities. Let q_{im} denote the quantity of commodity *m* purchased at time t_i and let $\mathbf{q_i}$ denote the vector of quantities at time t_i. Obviously we must have

$$(21) \qquad b_i = \sum_m p_{im} q_{im} > 0$$

with the constraints

$$(22) \qquad q_{im} \geq 0 \qquad \text{for all } i \text{ and for all } m$$

which, in view of (21), implies that at least one of the q_{im} is strictly greater than zero. More succinctly,

$$(23) \qquad b_i = \mathbf{p_i} \bullet \mathbf{q_i}$$

$$(24) \qquad \mathbf{q_i} > \mathbf{0}$$

where we have used the notation "$\mathbf{x} \bullet \mathbf{y}$" to indicate the scalar product of the vectors $\mathbf{x}$ and $\mathbf{y}$ and the notation "$\mathbf{x} > \mathbf{0}$" to indicate that each component of the vector $\mathbf{x}$ is non-negative *and* that at least one of its component is strictly greater than zero.

We assume that there is a *utility function* $U(\mathbf{q})$ that determines for each individual the "level of satisfaction" achieved by the individual as a function of the quantity vector $\mathbf{q}$ purchased by the individual. We will assume for the time being that the function does not depend explicitly on time. We assume that the function $U(\mathbf{q})$ is a non-negative real valued function and that its domain is the set of all vectors $\mathbf{q}$ such that $\mathbf{q} \geq \mathbf{0}$. We will assume that the function $U(\mathbf{q})$ satisfies certain "regularity" properties, namely:

1. the function is everywhere continuous and differentiable with respect to every component of **q;**
2. for all values of $\mathbf{q}$ **:**

$$\frac{\partial U}{\partial \mathbf{q}} > \mathbf{0}$$

i.e. all first order derivatives are non-negative and at least one of them is strictly positive. These conditions maybe more stringent than strictly necessary, but this is not an issue, as it will become clearer in the following. The *utility* enjoyed by the purchaser at time t_i is given by

$$(25) \qquad u_i = U(\mathbf{q_i})$$

We assume that at any given time the purchaser *attempts* to acquire the various commodities in quantities such that the function u_i is maximized, subject to the conditions

$$\mathbf{p_i} \bullet \mathbf{q_i} = b_i \qquad \mathbf{q_i} > \mathbf{0} \tag{26}$$

Define the *optimum utility function* $U^*(b,\mathbf{p}\,|\,U)$ as

$$U^*(b,\mathbf{p}\,|\,U) = \max_{\mathbf{q}} \left\{U(\mathbf{q}) : \mathbf{p} \bullet \mathbf{q} = b, \mathbf{q} \geq 0\right\} \tag{27}$$

$U^*(b,\mathbf{p}\,|\,U)$ measures the *maximum* value of the utility function $U(\mathbf{q})$ that can be achieved by somebody with a budget b, given the price vector $\mathbf{p}$ and according to the utility function U**.** Given the regularity conditions we have assumed, the function $U^*(b,\mathbf{p}\,|\,U)$ is strictly monotonically increasing with respect to b, for any $\mathbf{p}$. Note that for any positive α

$$U^*(\alpha b,\alpha\mathbf{p}\,|\,U) = U^*(b,\mathbf{p}\,|\,U) \tag{28}$$

i.e. if both the price vector and the budget are multiplied by the same constant, the optimal value of the utility function remains invariant. Let's also define the *minimum budget function* $B^*(u,\mathbf{p}\,|\,U)$ as

$$B^*(u,\mathbf{p}\,|\,U) = \min_{\mathbf{q}} \left\{\mathbf{p} \bullet \mathbf{q} : U(\mathbf{q}) = u, \mathbf{q} \geq \mathbf{0}\right\} \tag{29}$$

In other words $B^*(u,\mathbf{p}\,|\,U)$ represents the *minimum* value of budget that is required, given the price vector $\mathbf{p}$, to purchase a quantity vector $\mathbf{q}$ that will achieve the value u for the utility function U. Given the regularity conditions that we have assumed, the function $B^*(u,\mathbf{p}\,|\,U)$ is strictly monotonically increasing with respect to u, for any $\mathbf{p}$. Note that for any positive α

$$B^*(u,\alpha\mathbf{p}\,|\,U) = \alpha B^*(u,\mathbf{p}\,|\,U) \tag{30}$$

i.e. if the price vector is multiplied by a positive number, the budget necessary to achieve a specific value of the utility function is also multiplied by the same constant. It should be obvious that the functions $U^*(b,\mathbf{p}\,|\,U)$ and $B^*(u,\mathbf{p}\,|\,U)$ are the inverse of each other, i.e.:

$$B^*(U^*(b,\mathbf{p}\,|\,U),\mathbf{p}\,|\,U) = b \tag{31}$$

$$U^*(B^*(u,\mathbf{p},U),\mathbf{p}\,|\,U) = u \tag{32}$$

We can define the *price index function* $I(\mathbf{p_x},\mathbf{p_y},u\,|\,U)$ for the price vector $\mathbf{p_x}$ relative to the price vector $\mathbf{p_y}$, at the utility level u, as

$$I(\mathbf{p_x},\mathbf{p_y},u\,|\,U) = \frac{B^*(u,\mathbf{p_x}\,|\,U)}{B^*(u,\mathbf{p_y}\,|\,U)} \tag{33}$$

The function $I(\mathbf{p_x},\mathbf{p_y},u\,|\,U)$ measures the ratio of the budgets required at the two specified price levels to achieve the same value u of the utility function, given an *optimal* selection of the corresponding quantity vectors. Note that for any $\mathbf{p_x}$, $\mathbf{p_y}$ and $\mathbf{p_z}$ we always have

(34) $$I(\mathbf{p_x},\mathbf{p_y},u\,|\,U)=I(\mathbf{p_x},\mathbf{p_z},u\,|\,U)I(\mathbf{p_z},\mathbf{p_y},u\,|\,U)$$

i.e. the *transitivity property* is always satisfied. Note also that

(35) $$I(\mathbf{p_x},\mathbf{p_y},u\,|\,U)=\frac{1}{I(\mathbf{p_y},\mathbf{p_x},u\,|\,U)}$$

showing that the defined function satisfies the *reciprocity property* at a *specific level of utility value u*. Finally we obviously have for all $\mathbf{p_x}$ and for all *u* that

(36) $$I(\mathbf{p_x},\mathbf{p_x},u)=1$$

implying that the value of the index for any two times at which the price vector is the same is always identically equal to 1, i.e. that the function $I(\mathbf{p_x},\mathbf{p_y},u\,|\,U)$ satisfies the *identity* property. Furthermore, the value of the index function does not depend in any way on the *times* at which the specified price vectors occur, but depends only on the value of those price vectors.

We can also define the *equivalent budget function* $B_e(\mathbf{p_x},\mathbf{p_y},b\,|\,U)$ as

(37) $$B_e(\mathbf{p_x},\mathbf{p_y},b\,|\,U)=B^*(U^*(b,\mathbf{p_y}\,|\,U),\mathbf{p_x}\,|\,U)$$

In other words, the function $B_e(\mathbf{p_x},\mathbf{p_y},b\,|\,U)$ measures the minimum budget that would be required, at the price vector $\mathbf{p_x}$, to achieve the same level of utility that could be *optimally* obtained with the budget *b* at the price vector $\mathbf{p_y}$, given the utility function ***U*.** Notice that in general we have no reason to expect that the function $B_e(\mathbf{p_x},\mathbf{p_y},b\,|\,U)$ is linear with respect to *b*, for a given pair of price vectors, $\mathbf{p_x}$ and $\mathbf{p_y}$. In other words, in general it will be the case that

(38) $$B_e(\mathbf{p_x},\mathbf{p_y},\alpha b\,|\,U)\neq\alpha B_e(\mathbf{p_x},\mathbf{p_y},b\,|\,U)$$

We will define also the *equivalent relative budget function* $B_{er}(\mathbf{p_x},\mathbf{p_y},b\,|\,U)$ as

(39) $$B_{er}(\mathbf{p_x},\mathbf{p_y},b\,|\,U)=B_e(\mathbf{p_x},\mathbf{p_y},b\,|\,U)/b$$

Obviously we have

(40) $$B_{er}(\mathbf{p_x},\mathbf{p_y},b\,|\,U)=I(\mathbf{p_x},\mathbf{p_y},U^*(b,\mathbf{p_y})\,|\,U)$$

Consider the case of two time intervals t_i and t_j, with the corresponding values of the budgets and price vectors, b_i, $\mathbf{p_i}$ and b_j, $\mathbf{p_j}$. Let

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(41) $$u_i = U^*(b_i, \mathbf{p_i} \mid U)$$

(42) $$u_j = U^*(b_j, \mathbf{p_j} \mid U)$$

(43) $$G_i = I(\mathbf{p_i}, \mathbf{p_j}, u_i \mid U)$$

(44) $$G_j = I(\mathbf{p_i}, \mathbf{p_j}, u_j \mid U)$$

In other words, G_i is the index evaluated at u_i, the (optimum) value of the utility at time t_i and G_j is the index evaluated at the (optimum) value of the utility function at time t_j. In general the two indexes will be different, unless it happens that $u_i = u_j$.

We should note here a clear analogy with the two components of the Fisher standard price Index, as discussed in Section I.. The Laspeyres index (that uses the quantities of the "base" time period) corresponds to the G_j index, while the Paasche index (that uses the quantities of the "target" time period) corresponds to the G_i. There is however a fundamental conceptual difference. In the case of the Fisher Index, the two components combine the *prices* at one time with the *quantities* at another. This has a fundamental weakness. If there is a drastic change in the price of a specific commodity (relative to the general change in price levels), the combination of the quantities purchased at one time at a relatively low price, with the high price at the other time, will produce an unreasonable effect, as already discussed in the first section. In the case of G_i and G_j, the two estimates for the relative value of money are based on attempting to achieve different values of the utility function. In each case, however, the prices at any one time are used to determine the *optimum choice of commodity quantities* for the different budget level. If there is a drastic increase in the relative price level of certain commodities, the optimization procedure will normally keep the quantity of those commodities at a lower level (possibly at zero) and will achieve the desired value of the utility function through a redistribution of the budget among the other commodities. In general there should be no significant distortion.

We could then define an *average* index $G(i, j)$ as the geometric average of the two indexes G_i and G_j i.e. as

(45) $$G(i, j) = \sqrt{G_i G_j}$$

that could be viewed as a "generalized" analog of the Fisher Index defined in equation (1).

We need here to open a parenthesis and discuss the traditional notion of "price index". The basic traditional *bilateral* problem has been to find a way of comparing the situation at two different times t_1 and t_2, under the assumption that at those times we have observed the price levels $\mathbf{p_1}$, $\mathbf{p}_2$ and the *actually purchased* quantity vectors $\mathbf{q_1}$, $\mathbf{q}_2$. It has been typically assumed that the "price index" for those two times "ought" to be a function not only of the price levels, i.e. of $\mathbf{p_1}$ and $\mathbf{p}_2$, but also of the *observed* quantity levels, $\mathbf{q_1}$ and $\mathbf{q}_2$. This is typically demonstrated by the definition of the Fisher standard price index, as given in equation (1). In the analysis we have pursued so far there has been no reference to the quantities actually purchased at any given time. However, we have used indirectly the notion of the quantities that *ought to have been purchased* in order to achieve the minimum budget at a given utility level or the maximum utility at a given budget level.

Traditionally, in most analyses based on an underlying utility function, it has been assumed that the quantities purchased at any given price level would be the ones that *actually achieved the maximum value* of the underlying utility function. We said earlier that we assume that the purchaser of goods and services *attempts* to maximize the utility function, not that it actually *succeeds*. What would stop our purchaser from achieving the desired optimization? There are at least two different perspectives.

The first is that in reality the purchaser does not operate in a market of unlimited and unconstrained resources. The desire of the purchaser to acquire a certain quantity of a certain commodity may be limited by the actual availability of that commodity. Also, it is possible that the acquisition of a certain amount of a given commodity may be linked to the purchase of some other commodity, therefore constraining the relative ratios in which certain commodities may be purchased. An even more important issue is that a purchaser does not typically observe *all* prices before making any single acquisition, a procedure that would be required in order to guarantee that the optimum quantity of each commodity is actually selected. Finally, the purchaser may make random mistakes.

But the more fundamental issue, from our perspective, is that the assumption of a purchaser behavior based on the maximization of a utility function is only a *convenient mathematical framework*. The purpose of the model is to *approximate* as closely as possible the actual observed behavior. No one really believes that a real life purchaser would go around measuring all available prices and then would sit down with a computer to determine what quantities should be bought. To require an *exact match* between the values of the *actually purchased* quantities and the values predicted by the model is unnecessary.

III. The Basic Model

All of the above is rather straightforward and was presented mainly to establish the conceptual and notational framework.. In order to transform the above set of definitions into a workable procedure we must choose a specific expression for the function $U(\mathbf{q})$. Whenever one has to select a specific mathematical model to represent a class of real world phenomena it is rather common to proceed in two phases. In the first phase we select a *class* of mathematical models that we believe to be appropriate for the specific phenomena. Such class is typically characterized by a vector of parameters. In the second phase we proceed to select a specific value of the parameter vector on the basis of observed data.

We assume that the function is $U(\mathbf{q})$ of the form

$$U(\mathbf{q}) = \sum_m \frac{a_m c_m q_m}{c_m + q_m} \tag{46}$$

where the a_m and c_m are constant parameters associated with each commodity *m* satisfying the conditions

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(47) $$a_m > 0 \quad \text{and} \quad c_m > 0 \qquad \text{for all } m$$

We will use the vector notation “**a**” and “**c**” to indicate the vectors whose components are the a_m and c_m parameters respectively. Note that the vectors **a**, **c**, **p** and **q** have all the same dimensionality, i.e. the number M of commodities. The function *U* is obviously completely characterized by the pair of vectors, **a** and **c**. Note that the function U is continuous, differentiable and that for all values of **q,**

(48) $$\frac{\partial U}{\partial q_m} = \frac{a_m c_m^2}{(c_m + q_m)^2} > 0$$

The function $U(\mathbf{q})$ achieves its finite maximum of

(49) $$\sum_m a_m c_m$$

when the quantities are infinite. Since the maximization of the function $U(\mathbf{q})$ is unaffected by multiplying it by a constant positive factor, we can assume, without any loss of generality, that the function *U* is *normalized* so that

(50) $$\sum_m a_m c_m = 1$$

or, more concisely,

(51) $$\mathbf{a} \bullet \mathbf{c} = 1$$

This implies that when the values of the quantities q_m are *infinite*, the function *U* achieves its maximum value of 1. What is the “meaning” of the constants a_m’s and c_m’s? The maximum contribution to the function $U(\mathbf{q})$ that any given commodity *m* may provide, when it is infinite, is obviously $a_m c_m$. If $q_m = c_m$, the contribution of that particular commodity to the overall value of the function will be $a_m c_m / 2$, i.e. exactly one half of its possible maximum. When the value of q_m is small compared with c_m, the contribution due to commodity *m* will be approximately $a_m q_m$, i.e. the parameter a_m represents the initial value of the derivative of $U(\mathbf{q})$ with respect to q_m, i.e. the initial rate of “desirability” for commodity *m.*

Before we continue we need to discuss the “reasonableness” of the assumptions we just made. The idea that the “consumer” behaves so as to maximize some kind of “utility function” is obviously an old one. Is our specific choice of function a reasonable one? Strangely enough, it has been common in the economic literature on the subject of price indexes to talk about utility functions that are *linear* with respect to the vector **q**, i.e. it has often been assumed that, for any positive constant α

(52) $$U(\alpha \mathbf{q}) = \alpha U(\mathbf{q})$$

This appears to us to be extremely strange. The most "natural" assumption about a "reasonable" utility function would appear to be that the incremental value of increasing the value of all quantities by a given amount should be a *decreasing function* of the value of the quantities. By analogy with a different class of economic analysis, it would appear reasonable to presume a "law of diminishing utility", conceptually similar to the well-known "law of diminishing returns". If the "utility" of owning a car is *x*, it is difficult to believe that the "utility" of owning 100 cars is 100*x*. Our choice of functional class is such that the derivative with respect to q_m is monotonically decreasing with respect to q_m. This is not necessarily a strong requirement. It is conceivable that for some commodities the derivative with respect to q_m could be *increasing* for "small" values of q_m; however, it would appear that for "large" values of q_m, the derivative "ought" to be decreasing as q_m increases. Our choice of function allows for *unlimited substitutability*, i.e. any commodity can be substituted for another in order to achieve a given value of $U(\mathbf{q})$. It is quite clear that some level of substitutability is a necessary characteristic of any acceptable utility function. It may be argued that *too much substitutability* is not appropriate, since in practice some commodities can only be substituted by a restricted set of other commodities. However, we believe that there must be some compromise between simplicity and accuracy and that our choice is an adequate one for the purpose at hand.

Let

(53) $$H(\mathbf{q},\mathbf{p},b) = U(\mathbf{q}) - \lambda^2 \left(\sum_m p_m q_m - b \right)$$

The optimal choice for the q_m's are the values that satisfy the relations

(54) $$\frac{\partial H(\mathbf{q},\mathbf{p},b)}{\partial q_m} = \frac{\partial U(\mathbf{q})}{\partial q_m} - \lambda^2 p_m = 0$$

subject to the condition $\mathbf{q} > \mathbf{0}$. This implies that the optimum quantities q_m^* must satisfy the relations

(55) $$\frac{a_m c_m^2}{(c_m + q_m^*)^2} = \lambda^2 p_m$$

from which we can derive the optimum values of the quantities to be

(56) $$q_m^*(b,\mathbf{p} \mid \mathbf{a},\mathbf{c}) = c_m \left[\frac{1}{\lambda(b)} \sqrt{\frac{a_m}{p_m}} - 1 \right]$$

where

(57) $$\lambda(b)=\frac{\sum_{m} c_m\sqrt{a_m p_m}}{b+\sum_{m} c_m p_m}$$

in order to satisfy the condition

(58) $$\mathbf{p}\bullet\mathbf{q}=b$$

However, it may result that the quantities $q_m^*(b,\mathbf{p}\,|\,\mathbf{a},\mathbf{c})$ from equation (56) are negative. This obviously violates condition the condition $\mathbf{q}>\mathbf{0}$. The solution is that the quantities of such commodities must be set to zero and the corresponding index must be removed from the summation of equation (57). Note that, for any positive α we will have

(59) $$q_m^*(\alpha b,\alpha\mathbf{p}\,|\,\mathbf{a},\mathbf{c})=q_m^*(b,\mathbf{p}\,|\,\mathbf{a},\mathbf{c})$$

In other words, if all prices changes at the same rate, the value of the quantities will remain the same *if* the budget is also increased by the same factor. By substituting from (56) and (57) into (46) we can derive the form of the function $U^*(b,\mathbf{p}\,|\,\mathbf{a},\mathbf{c})$, corresponding to definition (27), as

(60) $$U^*(b,\mathbf{p}\,|\,\mathbf{a},\mathbf{c})=\sum_{m} a_m c_m-\frac{\left(\sum_{m} c_n\sqrt{a_m p_m}\right)^2}{b+\sum_{m} c_m p_m}$$

However, it should be remembered that all summations in equation (60) must be extended *only* to those commodities *m* for which equation (56) leads to non-negative values for the quantities $q_m^*(b,\mathbf{p}\,|\,\mathbf{a},\mathbf{c})$. We can also *formally* solve for the function $B^*(u,\mathbf{p}\,|\,\mathbf{a},\mathbf{c})$, corresponding to equation (29), as follows

(61) $$B^*(u,\mathbf{p}\,|\,\mathbf{a},\mathbf{c})=\frac{\left(\sum_{m} c_m\sqrt{a_m p_m}\right)^2}{\sum_{m} a_m c_m-u}-\sum_{m} c_m p_m$$

We must remember again that all the summations must be extended *only* to those commodities for which equation (56) provides a non-negative result. Since this depends on the value of the budget, *b*, for which the quantities are estimated, the value of the function $B^*(u,\mathbf{p}\,|\,U)$ must be determined iteratively. In practice, the value given by equation (61), when the summations are extended to all commodities, provides a very good initial estimate. Typically, for large values of *b*, which correspond to large values of *u*, all commodities will have non-zero quantity values and equations (60) and (61) can be evaluated straightforwardly. However, at the other extreme, for very small values of *b*, which correspond to low values of *u*, the situation is quite different. For values of *b* near zero, only the commodity with the most favorable relation between initial "desirability" and "price" will have a non-zero quantity. This is determined by the factor

$$\frac{a_m}{p_m} \tag{62}$$

having the highest value.

Assume that at some time t_i we know the values of b_i, of the prices vector $\mathbf{p_i}$ and of the quantity vector $\mathbf{q_i}$. Can we solve for $\mathbf{a}$ and $\mathbf{c}$? The answer is "yes", but not uniquely. The function $U(\mathbf{q})$ is characterized by 2M-1 parameters (since equation (50) removes one degree of freedom). However, at each time t_i we only have M-1 independent constraints, since condition (58) also removes one degree of freedom. If we know the values b_i, $\mathbf{p_i}$, $\mathbf{q_i}$ and b_j, $\mathbf{p_j}$, $\mathbf{q_j}$ for *two* time intervals t_i and t_j, we can then "almost" solve for the $\mathbf{a}$ and $\mathbf{c}$, there being only one degree of freedom left. However, it is not true that we can find a solution for *any possible combination* of the above observed values. There are certain combinations that cannot be achieved for *any* selection of **a** and **c**. But finding a "solution" for **a** and **c** in such a way is not what we will try to do. As we mentioned earlier, we assume that the purchaser *attempts* to maximize the value of the utility function $U(\mathbf{q})$, but *not* that it actually *achieves* it. This means that we do not necessarily assume that the quantity vector $\mathbf{q_i}$ *actually acquired* by the purchaser at time t_i actually achieves the optimum value of the utility function, as determined by equation (60), for the appropriate value of the purchaser budget, b_i. What we *do* assume is that the purchaser selects the quantities of all commodities in such a way as to approximate as much as possible the optimal values appropriate to its budget and according to its utility function. In order to translate this assertion into an evaluation procedure for the parameter vectors, we can define an *error function* $S(\mathbf{a},\mathbf{c})$ that measures the "error" between the actual value of the quantity vector and its optimal value. There are a number of reasonable choices for such error function. We have chosen to use the one defined by equation (63), namely

$$S(\mathbf{a},\mathbf{c}) = \sum_{i=1}^{N} \frac{1}{b_i^2} \left[\sum_{m=1}^{M} \left[p_{im} \left[q_{im} - q_m^*\left(b_i, \mathbf{p_i} \mid \mathbf{a},\mathbf{c}\right) \right] \right]^2 \right] \tag{63}$$

where the sum on *i* are extended over all times t_i for which data is available**.** The function $S(\mathbf{a},\mathbf{c})$ essentially measures the difference between the actual quantities purchased and the optimum quantities that would have been purchased, with the specified budget and the given price vector, if the given utility function would have been applied. The differences are weighed according to their percentage contribution to the total budget. Minimizing the function $S(\mathbf{a},\mathbf{c})$ is equivalent to attempt to find the pair of vectors **a** and **c** that will define a utility function that will determine a selection of quantities to be purchased that is as close as possible to those actually purchased by the consumer.

The problem of the actual minimization of the function $S(\mathbf{a},\mathbf{c})$ is a non-trivial one. The complexity of the function is such that there is no obvious way to solve the minimization problem algebraically. In any specific case, it is of course possible to solve the problem

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numerically, by a standard gradient procedure. There are, however, a number of practical problems. The complexity of the numerical computation to evaluate the value of the function $S(\mathbf{a},\mathbf{c})$ and of its gradient relative to **a** and **c** is approximately proportional to NM^2. In most interesting cases the value of M may be over 100. This will make the amount of computation non trivial. But this is not the most difficult issue. The major problem is that the minimization of a function of 2M-1 variables, with M in the hundreds, presents a number of pitfalls. A normal gradient procedure will lead to a *local minimum*, which is a function of the choice of the initial starting point. Any attempt to systematically search for a *global minimum* would entail a number of function evaluations of the order of $2^{(2M-1)}$. For M in the hundreds, this is impractical. The choice of "good" starting points and the decision to stop searching will have to be based on pragmatic considerations.

IV. The Choice of "Reference" Utility Values

Let's assume that we have chosen a pair of vectors, $\mathbf{a^o}$ and $\mathbf{c^o}$ which define our assumed utility function. We can now evaluate the price index function $I(\mathbf{p_x},\mathbf{p_y},u\,|\,U)$ for any pair of price vectors. However, we have to determine at what value *u* for the utility function we are going to make the evaluation, or, if necessary, how we should combine evaluations made at different utility levels. In Section II we introduced the two indexes, G_i and G_j that would appear to be good candidates if we had to compare only two price vectors (the *bilateral* problem). However, our approach must be able to be generalized to the case of an arbitrary long sequence of price vectors (the *multilateral* problem).

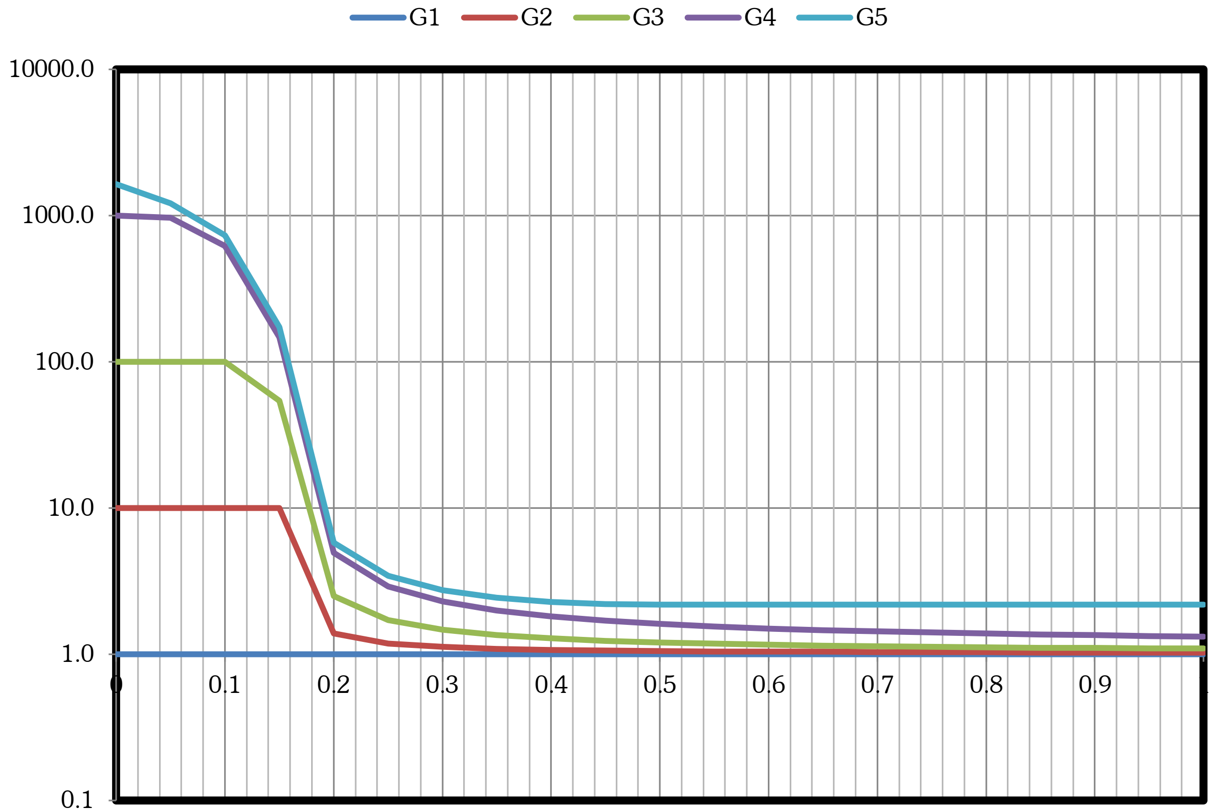
The importance of the transitivity property for the price indexes can be clearly understood when looking at the problem of dealing with N time intervals, t_i, with i = 1, 2, …., N. Without the transitivity property we would have to deal with N(N-1)/2 separate binary comparisons (assuming that at least the reciprocity property holds!). If the transitivity property holds, we can select an arbitrary time interval, say t_r, as the *reference time interval* and then only deal with the N index functions $I(\mathbf{p_i},\mathbf{p_r},u\,|\,U)$. Interestingly enough, we could actually select an arbitrary *reference price vector*, $\mathbf{p_r}$, even although it may not be associated with *any* time interval. Such generalization, however, appears to be more confusing than useful, unless there was some significant justification for a "special" price vector.

In order to understand better the dependence on the choice of the value *u*, at which to determine the price index, we will return to our deliberately extreme example of section I. If we apply our approach to the situation described by equations (11)-(15) we find that a "good" choice for the parameter vectors **a** and **c** are

(64) $$\mathbf{a^o} = (0.2317, 383.0719)$$

(65) $$\mathbf{c^o} = (3.60504, 0.00043)$$

Fig. 1 Index Values as a function of utility value



and that the *optimal* values of the utility function associated with the five time points are:

(66) $$u_1 = 0.4576 \quad u_2 = 0.4466 \quad u_3 = 0.4139 \quad u_4 = 0.3363 \quad u_5 = 0.2980$$

We will choose t_1 as our reference time interval.

In Fig 1 we show the value of the *price index functions* $G_i(u)$ where

(67) $$G_i(u) = I(\mathbf{p_i}, \mathbf{p_1}, u \mid \mathbf{a^o}, \mathbf{c^o})$$

Note that index values are shown on a logarithmic scale in order to demonstrate the large differences between the values for small values of *u* and large values of *u*.

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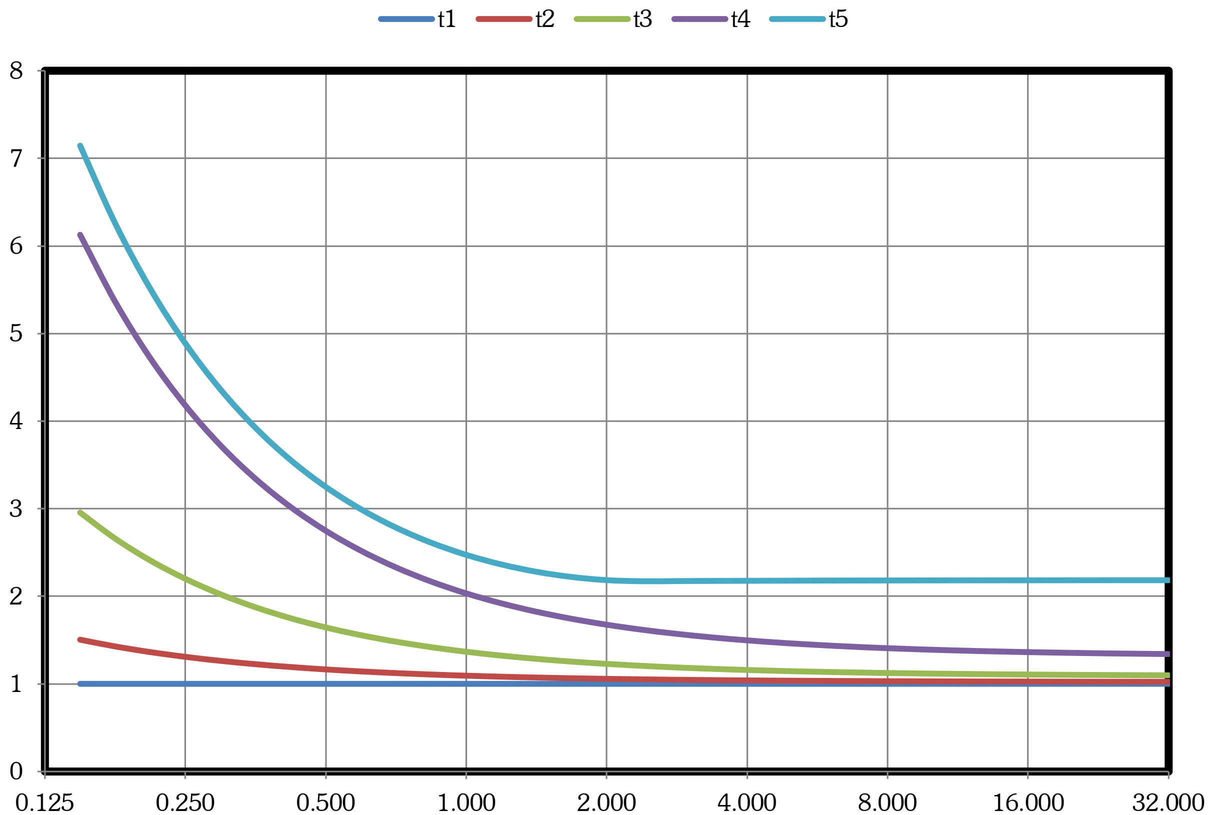
Fig. 2 Equivalent Budget ratio at different time points



The same basic information is shown in Fig 2 from a different perspective, i.e. as the *equivalent relative budget functions* $B_i(b)$ where

$$(68) \qquad B_i(b) = B_{er}(\mathbf{p_i}, \mathbf{p_1}, b \mid \mathbf{a^o}, \mathbf{c^o})$$

Both curves show that for low values of *u* and *b* (which are related), the price index and the relative equivalent budget are very high, but that they both converge toward much lower values for higher values of *u* and *b*. The two sets of curves clearly show that the "value of money" at any given time is not a constant, but a function of the budget of the purchaser.

Let's now go back to the general case. Can we "fairly" summarize the situation with a *single price index value*? Quite obviously we can define an "average value of money' in a variety of ways. For example, let

$$(69) \qquad u_i^* = U^*(b_i, \mathbf{p_i} \mid \mathbf{a^o}, \mathbf{c^o})$$

be the optimal value of the utility function that would have been achieved at time t_i according to the assumed choice of parameter vectors $\mathbf{a^o}$ and $\mathbf{c^o}$. Define the *(geometric) average utility* $\bar{u}^*$ as

(70) $$\bar{u}^* = \sqrt[N]{\prod_{i=1}^{N} u_i^*}$$

We can then define the price index $G_1(i, j)$ as

(71) $$G_1(i, j) = I(\mathbf{p_i}, \mathbf{p_j}, \bar{u}^* \mid \mathbf{a^o}, \mathbf{c^o})$$

The price index $G_1(i, j)$ essentially measures the price index at the utility level that corresponds to the geometric average of the utility values that would have been achieved in each time period if the budget available at each time period were optimally utilized, according to the specified choice of parameter vectors for the utility function.

Another approach is to define the index $G_2(i, j)$ as

(72) $$G_2(i, j) = \sqrt[N]{\prod_{k=1}^{N} I(\mathbf{p_i}, \mathbf{p_j}, u_k^* \mid \mathbf{a^o}, \mathbf{c^o})}$$

In the case of the index $G_1(i, j)$ we have taken the geometric average of the utility values and then evaluated the index at that average. In the case of the index $G_2(i, j)$ we have evaluated the indices for all optimal values of the utility function and then we have taken the geometric average of the result. The choice of the *geometric* average for the index $G_2(i, j)$ is suggested by the fact that in this way the *transitivity*, *reciprocity* and *identity* properties will all remain valid. In the case of the index $G_1(i, j)$ this would occur no matter what kind of average we would have chosen. The selection of the geometric average for the $G_1(i, j)$ price index is purely for symmetry with respect to the index $G_2(i, j)$.

In addition to some kind of averaging of the data, there is another approach that also merits attention. This is to select one *fixed* value of *u*. The selection of a predetermined value for *u* has an advantage, namely that the price index values do not have to be reevaluated as new data becomes available, for additional time intervals. An interesting choice is to select

(73) $$u = 0.5$$

that corresponds to half of the maximum value of the utility function. Another interesting choice could be to select

(74) $$u = 1$$

i.e. to measure the price index at the "ultimate' value of the utility function. In the latter case it should be noted that the function $B^*(u, \mathbf{p} \mid \mathbf{a}, \mathbf{c})$ goes to infinity as *u* approaches 1. However, the value of the ratio defined by equation (33) can be evaluated by standard limiting procedure to be

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(75) $$\lim_{u\to 1} I(\mathbf{p_x}, \mathbf{p_y}, u \mid \mathbf{a}, \mathbf{c}) = \left[\frac{\sum_m c_m \sqrt{a_m p_{xm}}}{\sum_m c_m \sqrt{a_m p_{ym}}} \right]^2$$

However, the choice of such a limiting case will lead to possibly anomalous results in the presence of very large changes in commodity prices. More generally, we can choose an arbitrary set of values of u and then perform the geometric averaging of the results, as we have done in equation (72). Let

(76) $$\{v_1, v_2, ..., v_n\}$$

be a set of n constants where

(77) $$n \geq 1; \quad 0 < v_k < 1 \quad \text{for all } k$$

We can define the *generalized price index function* $G(i, j \mid v_1, v_2, ..., v_n)$ as

(78) $$G(i, j \mid v_1, ..., v_n) = \sqrt[n]{\prod_{k=1}^{n} I(\mathbf{p_i}, \mathbf{p_j}, v_k \mid \mathbf{a^o}, \mathbf{c^o})}$$

We will define the following particular choices of indexes:

(79) $$G_1(i, j) = G(i, j \mid \bar{u}^*)$$
(80) $$G_2(i, j) = G(i, j \mid u_1^*, u_2^*, ..., u_N^*)$$
(81) $$G_3(i, j) = G(i, j \mid 0.5)$$

Note that the G_1 and G_2 indexes correspond to the same indexes defined in equations (71) and (72).

	i =1, j =1	**i = 2, j = 1**	**i =3, j = 1**	**i = 4, j = 1**	**i = 5, j = 1**
$F(i, j)$	**1.00**	**2.66**	**7.49**	**22.95**	**70.71**
$F_c(i, j)$	**1.00**	**2.66**	**5.03**	**7.10**	**8.55**
$G_1(i, j)$	**1.00**	**1.07**	**1.30**	**1.85**	**2.30**
$G_2(i, j)$	**1.00**	**1.08**	**1.31**	**1.88**	**2.36**
$G_3(i, j)$	**1.00**	**1.05**	**1.20**	**1.61**	**2.17**

Table 1.

If we apply the above choices to our numerical example we will obtain the numbers in Table 1 where we have also added the corresponding values of the Fisher standard index and Fisher chain index. It is clear that the G_1 and G_2 indexes lead to numbers that (at least in this example) are almost identical. Since the value of $\bar{u}^*$ is approximately 0.39, it is not surprising that also the G_3 index (corresponding to $u = 0.5$) is very close. The most interesting fact is however, that all indexes appear to be much more "reasonable" from an intuitive point of view than either the Fisher standard or chain indexes (although the author intuition might be biased).

V. A Real Example

The trivial example we have been using up to now was meant only to demonstrate a specific difference between our proposed methodology and the classical Fisher approach. We will now look at a real life example in order to have a more valid comparison.

In all of the previous discussion we referred to a single individual purchaser. Unfortunately most of the economic data that we have is *aggregate* data about the overall purchases of all of the US residents. In order to be able to use that data in our analysis we will make the following simplifying assumptions:

- every US purchaser behaves according to the same utility function;
- the overall behavior of the US residents as a whole can be evaluated as if each resident had exactly the same budget, equal to the average budget obtained by dividing the overall budget for the US by the resident population of the US.

This is clearly an oversimplification, but it allows us to analyze some of the available data in a simple way. In order to test the approach in a real life situation we decided to apply it to the US data for the period 1950–2010. The data we used is that released by the Bureau of Economic Analysis (BEA) of the US Department of Commerce, as of June 24, 2011. More precisely, we used the annual BEA Tables labeled "Table 1.5.5 Gross Domestic Product, Expanded Detail" and "Table 1.5.4 Price Indexes for Domestic product, Expanded Detail" in the format made available by the BEA through their Internet website.

These tables provide the current dollar value of a number of line items and the *relative price* of those items with respect to the *average* of 2005. It is not possible to derive from those tables the *absolute quantities* of the items in question, but only the *relative quantities* with respect to the 2005 average. This is not a major obstacle, since it only changes the units in which all quantity variables are expressed, without affecting the substance of the analysis. The tables are expressed in units of billion US dollars, with the precision of a tenth of a billion. Because some of the partial aggregate figures were derived from the original (more accurate) data, we restated the total so as to match the available resolution. The data from the tables were modified to be consistent with our model, which is defined as a "purchaser behavior model". We define the **Gross Domestic Activity (GDA)** as the Gross Domestic Product with the following modifications:

- it adds Imports
- it subtracts Exports
- it subtracts any change in inventory

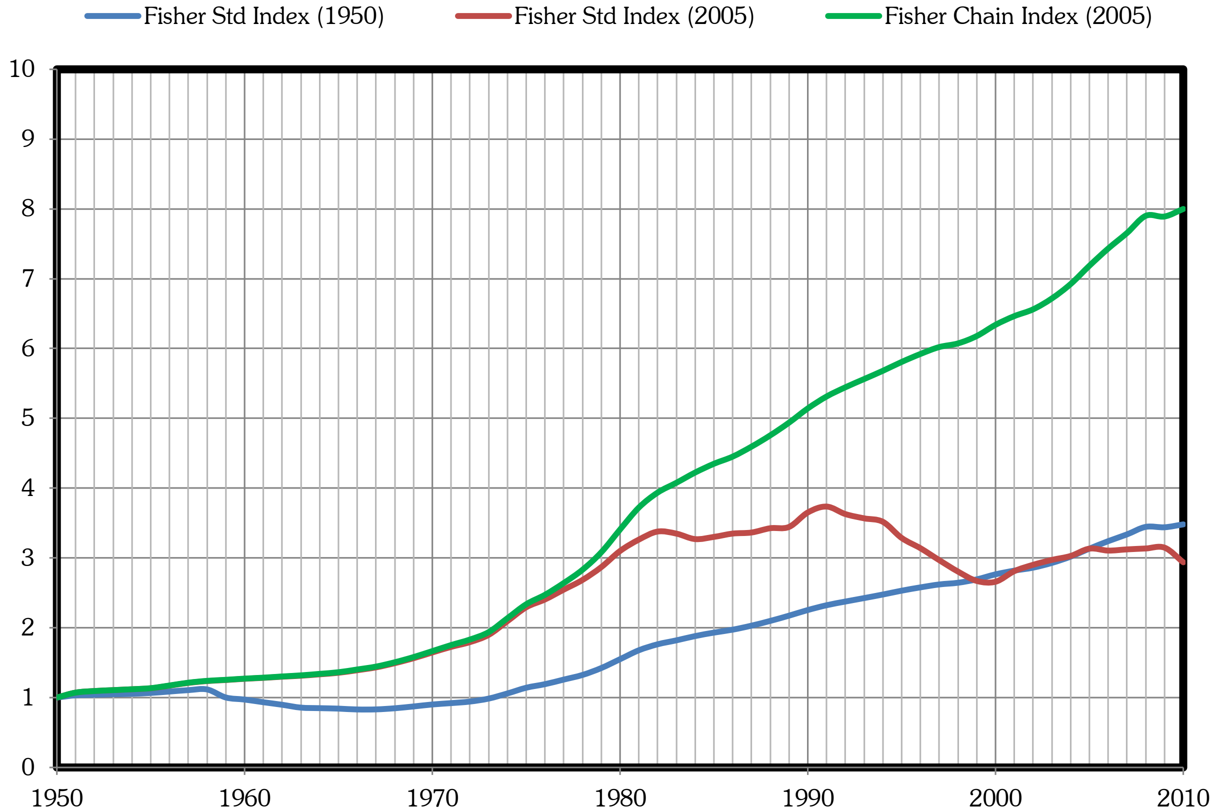
` ` `

The GDA measures the total amount of money that people have used to purchase goods and services (directly or indirectly) either for immediate consumption or to invest in new assets. With the above modifications, the table identifies the 30 commodities listed in Table 2 which we have used in our analysis. As mentioned above, the original price data is given relative to the year 2005. For our analysis it is preferable to use the year 1950 as the base year. This does not create any problems with either the Fisher chain index or our G indexes. However, as stated earlier, the Fisher standard index does depend on what year is used as the base year.

Table 2

1	Motor vehicles and parts
2	Furnishings and durable household equipment
3	Recreational goods and vehicles
4	Other durable goods
5	Food and beverages purchased for off-premises consumption
6	Clothing and footwear
7	Gasoline and other energy goods
8	Other nondurable goods
9	Housing and utilities
10	Health care
11	Transportation services
12	Recreation services
13	Food services and accommodations
14	Financial services and insurance
15	Other services
16	Final consumption expenditures of nonprofit institutions serving households
17	Private Investments: Structures
18	Private Investments: Computers and peripheral equipment
19	Private Investments: Software
20	Private Investments: Other office equipment
21	Private Investments: Industrial equipment
22	Private Investments: Transportation equipment
23	Private Investments: Other equipment
24	Private Investments: Residential
25	Federal Defense Consumption expenditures
26	Federal Defense Gross investment
27	Federal Civilian Consumption expenditures
28	Federal Civilian Gross investment
29	State and local Consumption expenditures
30	State and local Gross investment

Fig. 3 Comparison of Fisher Indexes



In Fig.3 we show

- the Fisher standard index using the year 1950 as base;
- the Fisher standard index using the year 2005 as base, renormalized so as to the set the value for 1950 to 1;
- the Fisher chain index using the year 1950 as base.
-

The difference between the Fisher standard indexes is quite significant, demonstrating the basic weakness of that index. The apparently "anomalous" behavior of the Fisher standard indexes is due almost exclusively to the presence of the commodity labeled "Private Investment: Computer and Peripheral Equipment". It is a well-known fact that the price of computer units has been going *down* in current $ terms while at the same time their "performance" has been increasing. The BEA has attempted to take this into account, according to the methodology normally referred as "hedonic indexes". Such methodology attempts to restate unit prices so as to take into consideration major changes in functionality. However, in so doing the BEA has gone overboard, overestimating the increased value of such commodity. This author has spent many years in the analysis of computer performance and price/performance, therefore he has some expertise in the subject. We have therefore restated the relative prices of the given commodity to be more in line with reality.

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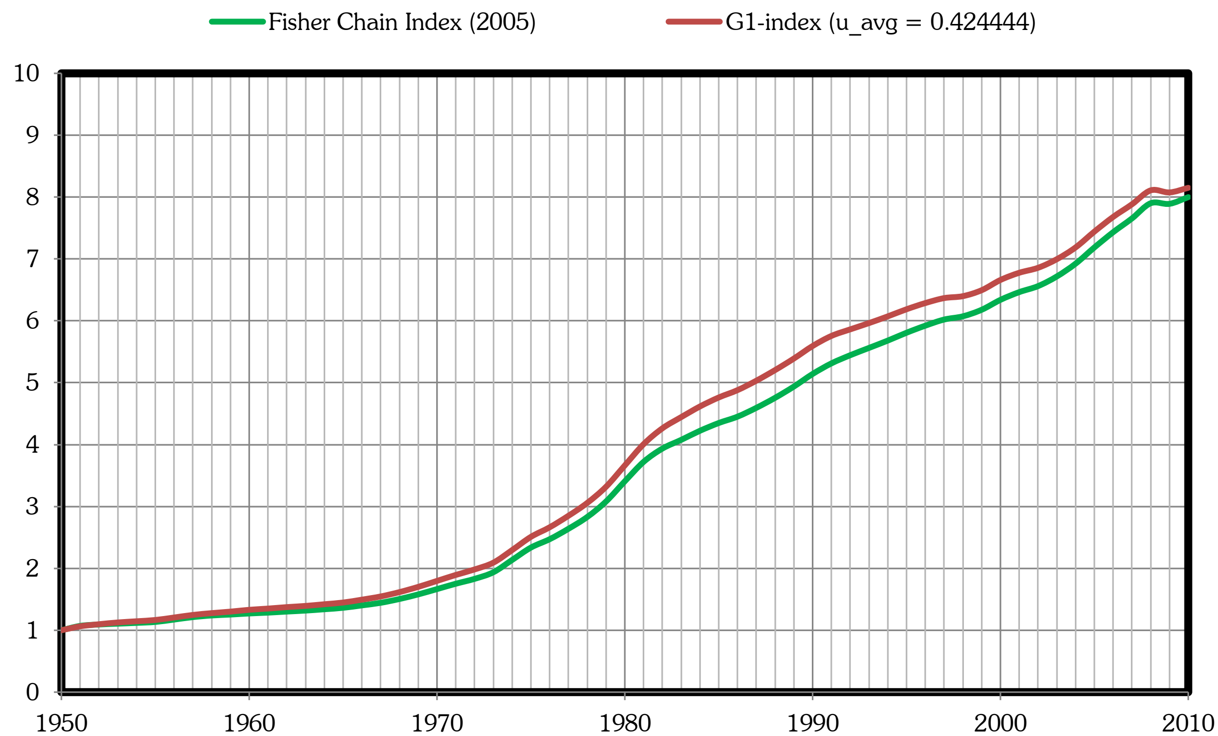
Fig. 4 Comparison between Fisher Chain Index and the G1 Index

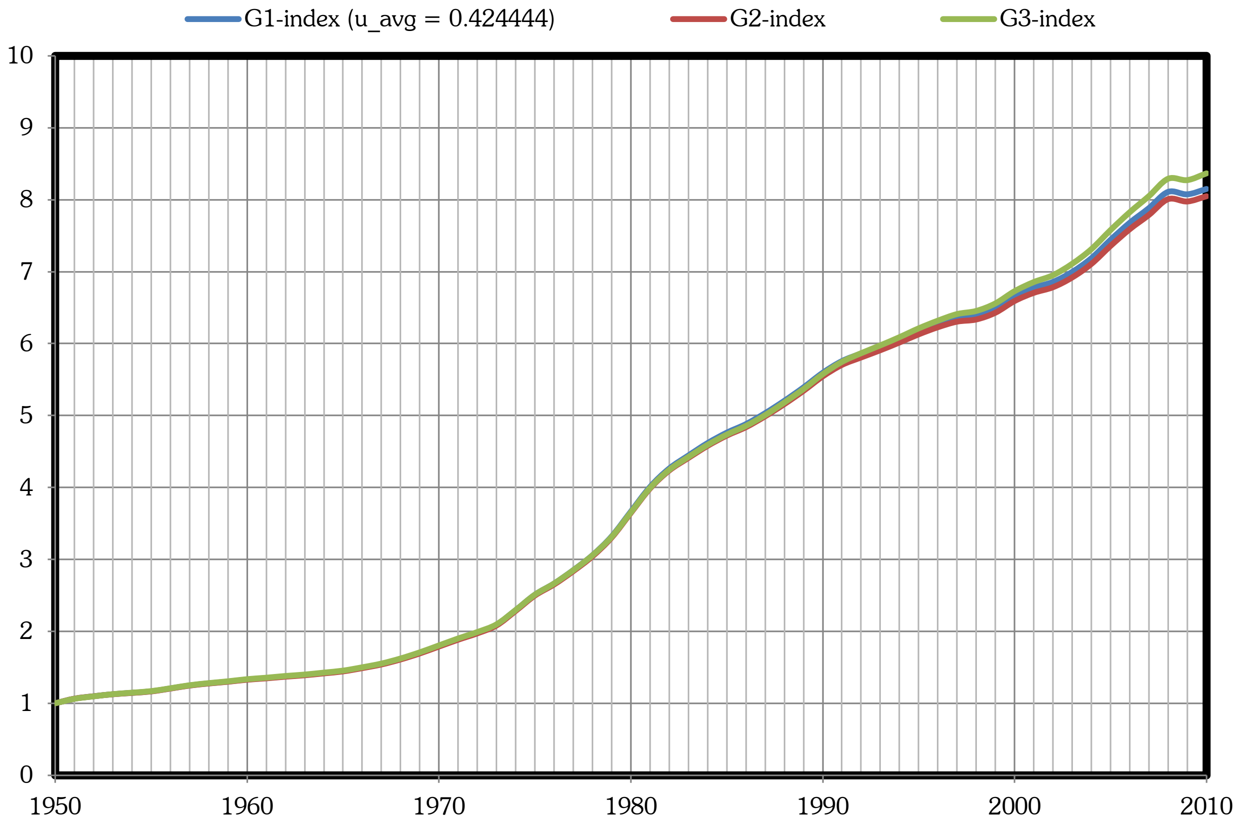


Even so, the still great historical improvement in the price/performance of computers and peripheral equipment distorts completely the Fisher indexes. Since the index uses the prices at one time in combination with the quantities at any other time, it creates an anomalously high evaluation of the index when comparing years at the two extremes of the time period in question.

In Fig.4 we show the Fisher chain index together with the G_1 index of our methodology. They show some significant similarity. This in a sense provides a "degree of confirmation" for the validity of both methodologies, since they achieve a high degree of mutual consistency, while approaching the problem from rather different points of view.

In Fig.5 we show all three of our indexes proposed above. They show remarkable similarity.

Fig.5 Comparison between different G Indexes



Our evaluations have been based on the behavior of an *average* consumer. We know of course that consumers at different income levels will behave differently. However, it is not unreasonable to presume that *all* consumers can be characterized by the same utility function, with the differences in consumption being determined only by the differences in budgets.

In Fig.6 we show the indexes relative to three choices of fixed values of u, namely

(82) $$u = 0.3 \quad u = 0.5 \quad u = 0.7$$

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**Fig. 6 Index function at different utility values**

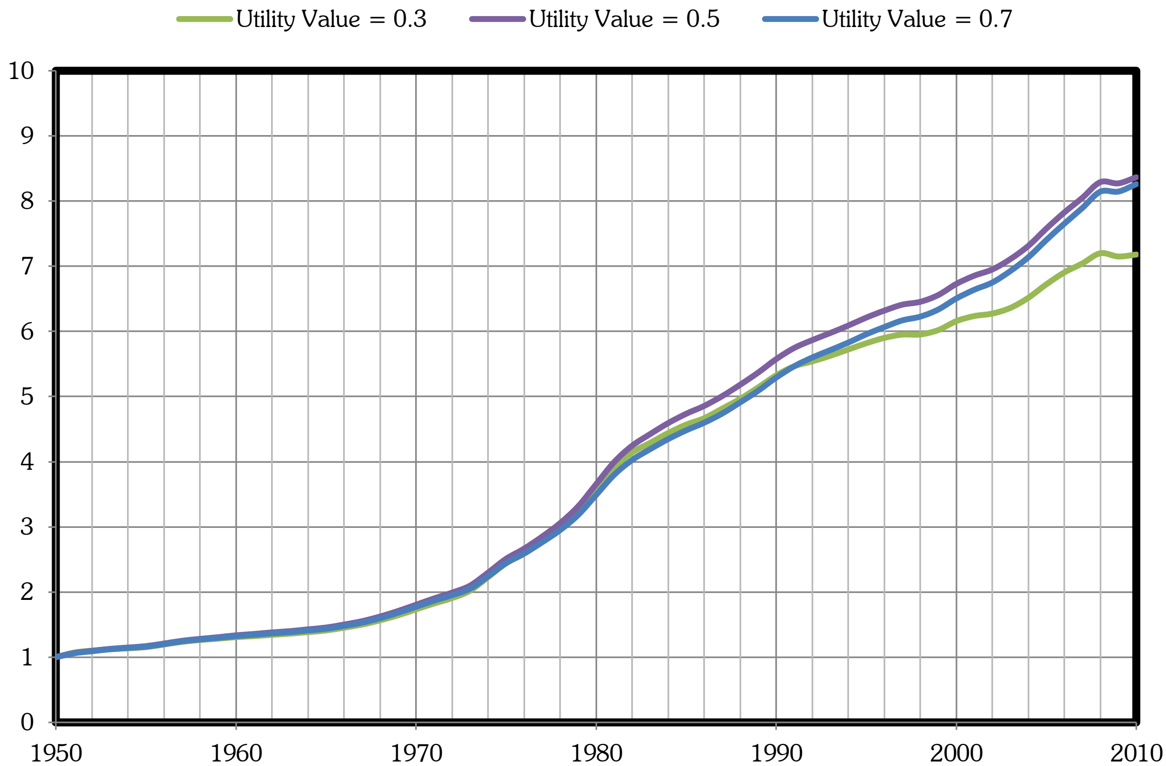
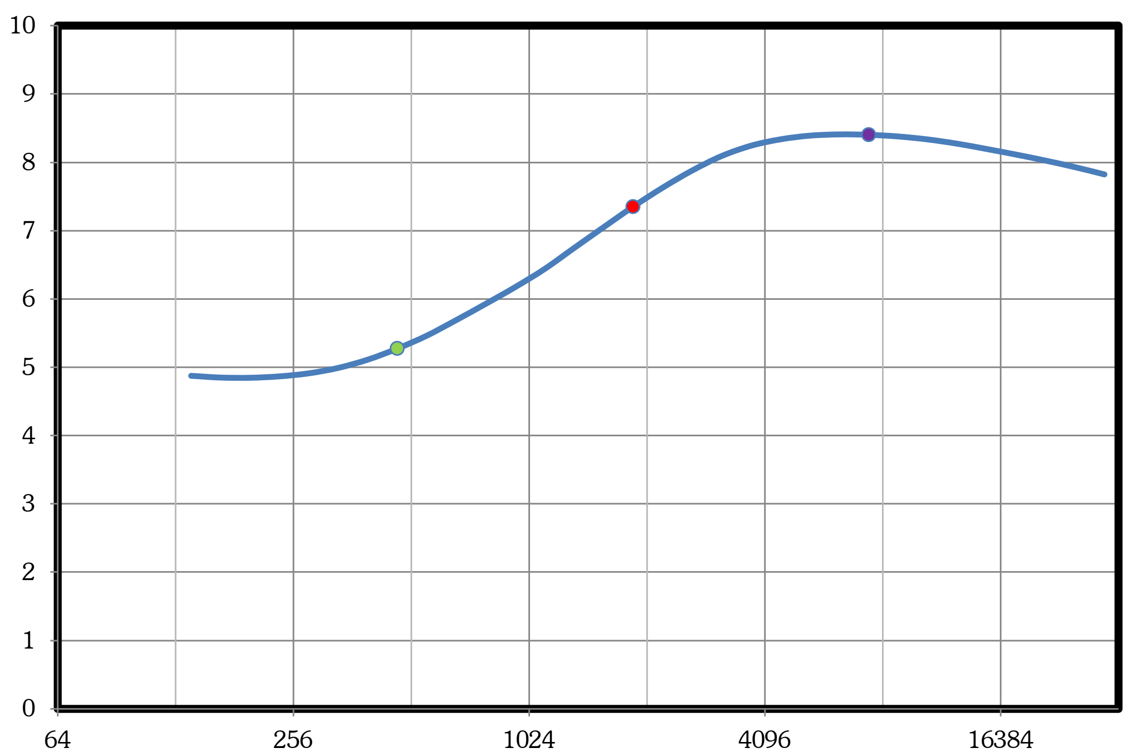



**Fig. 7 Equivalent 2010 budget relative to 1950 budget**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They may be assumed to approximately correspond to a relatively "low budget", "medium budget" and "high budget" consumer, since budget and utility values would be positively correlated with each other. If we limit ourselves to only look at the two extremes of the time period, i.e. the years 1950 and 2010, Fig.7 shows the equivalent budget ratio that would have been necessary to obtain in the year 2010 in order to achieve the same level of utility (in 2010) achieved by any given budget level in 1950. For the average budget (in 1950) the ratio is about 7.3, for a budget equal of one fourth of the average (in 1950), the ratio is about 5.2. For a budget equal to 4 times the average (in 1950) the ratio is about 8.4.

VI. Time Varying Utility Function

In the previous analysis we have assumed that the utility function is constant over time. When the overall time span is significant, this assumption may be questioned. We could approach the issue by hypothesizing a specific form of time dependence for the **a** and **c** vectors. Unfortunately the variety of reasonable options would lead us into a considerable level of complexity that may be inappropriate in the present context.

There is however a simple approach that can give us some insight into the issue. Assume we have N time intervals, t_i, with i = 1, 2, …., N. Let r be an integer, with $1 \le r \le N$. For each value of the index j , $1 \le j \le N$, define

(83) $$j_L = \max(0, j - r)$$

(84) $$j_H = \min(j + r, N)$$

In other words the interval (j_L, j_H) consists normally of $2r+1$ time points, centered around the time interval t_j, as long as such time points are available. Define the *local error function*

(85) $$S(j,\mathbf{a},\mathbf{c}) = \sum_{i=j_L}^{j_H} \frac{1}{b_i^2} \left[\sum_{m=1}^{M} \left[p_{im} \left[q_{im} - q_m^*\left(b_i, \mathbf{p_i} \,|\, \mathbf{a},\mathbf{c}\right) \right] \right]^2 \right]$$

which only takes into consideration the differences between the actual and optimal values of the quantity vectors for a limited number of time intervals, centered on the selected time interval. Define the *locally optimal vectors* $\mathbf{a^o}(j)$ and $\mathbf{b^o}(j)$ as those parameter vectors that minimize the function $S(j,\mathbf{a},\mathbf{c})$, *subject to the constraint*

(86) $$\mathbf{q_j} = \mathbf{q_j^*}\left(b_j, \mathbf{p_j} \,|\, \mathbf{a^o}(j), \mathbf{c^o}(j)\right)$$

i.e. subject to the constraint that for the specified index *j* the values of the *actual* quantity vector and of the *optimal* quantity vector are exactly the same. In other words, the utility function $U\left(\mathbf{q} \,|\, \mathbf{a^o}(j), \mathbf{c^o}(j)\right)$ is biased to match the actual purchasing behavior at time t_j, while minimizing the differences for the errors in neighboring time intervals. Let

```

(87) $$u_i^*(j) = U^*(b_i, \mathbf{p_i} \mid \mathbf{a^o}(j), \mathbf{c^o}(j))$$

and

(88) $$\bar{u}(j) = \sqrt[N]{\prod_{i=1}^{N} u_i^*(j)}$$

We can then define the analog indexes to the 3 introduced in equations (79) - (81), i.e.

(89) $$G_1(i, j \mid k, r) = I(\mathbf{p_i}, \mathbf{p_j}, \bar{u}^*(k) \mid \mathbf{a^o}(k), \mathbf{c^o}(k))$$

(90) $$G_2(i, j \mid k, r) = \sqrt[N]{\prod_{s=1}^{N} I(\mathbf{p_i}, \mathbf{p_j}, u_s^*(k) \mid \mathbf{a^o}(k), \mathbf{c^o}(k))}$$

(91) $$G_3(i, j \mid k, r) = I(\mathbf{p_i}, \mathbf{p_j}, 0.5 \mid \mathbf{a^o}(k), \mathbf{c^o}(k))$$

as the *locally optimal price indexes relative to the time interval k*.

We can apply the proposed approach to the example discussed in the previous section. We selected

(92) $$r = 10$$

for our analysis.

In order to make the figures somewhat more readable, we only show in Fig.8 the $G_3$ indexes for the years 1950, 1960, 1970,….. 2000, 2010. There is a noticeable systematic difference, with the utility functions optimized for later years indicating a higher rate of inflation. In this study we are concentrating on the methodology, so we will not attempt to analyze the reasons for the trend.

We can combine the locally optimal indexes into overall indexes

(93) $$\bar{G}_m(i, j \mid r) = \sqrt[n]{\prod_{k=1}^{n} G_m(i, j \mid k, r)} \qquad \text{for all } m$$

where $n$ is the number of time intervals for which the local optimization has been performed.
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Fig. 8 Yearly optimized indexes

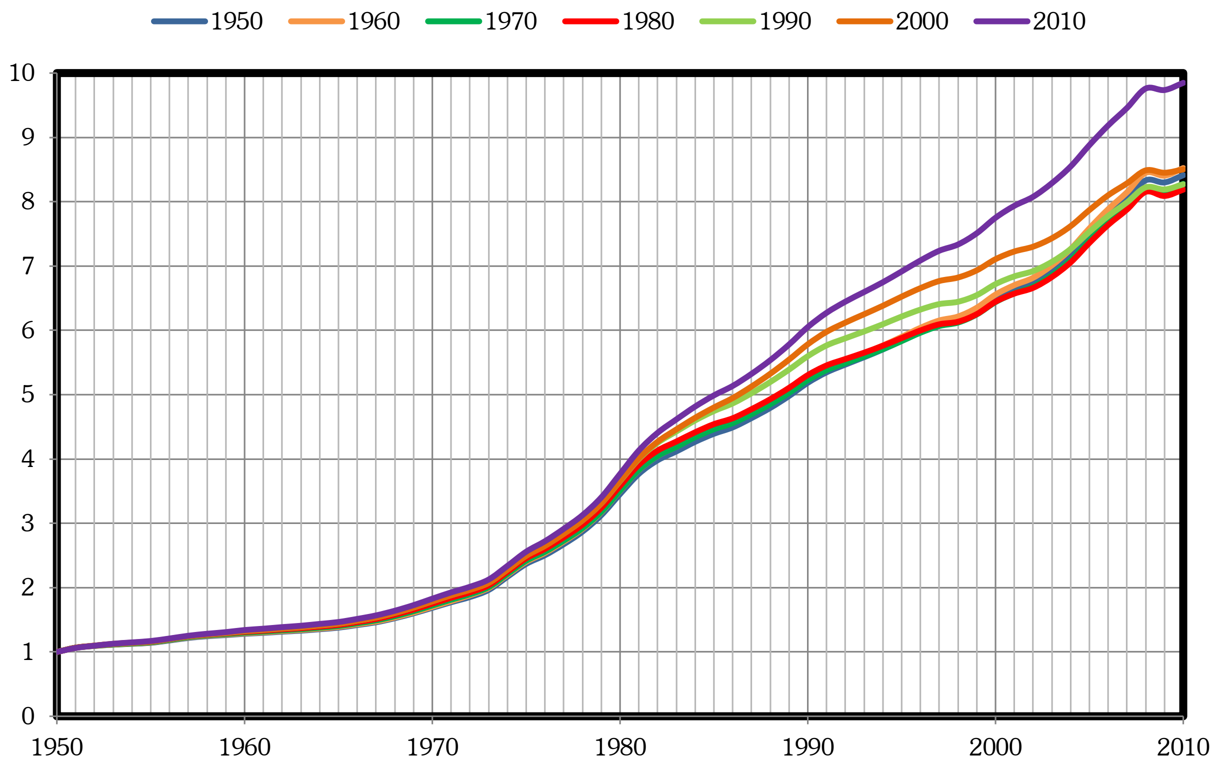
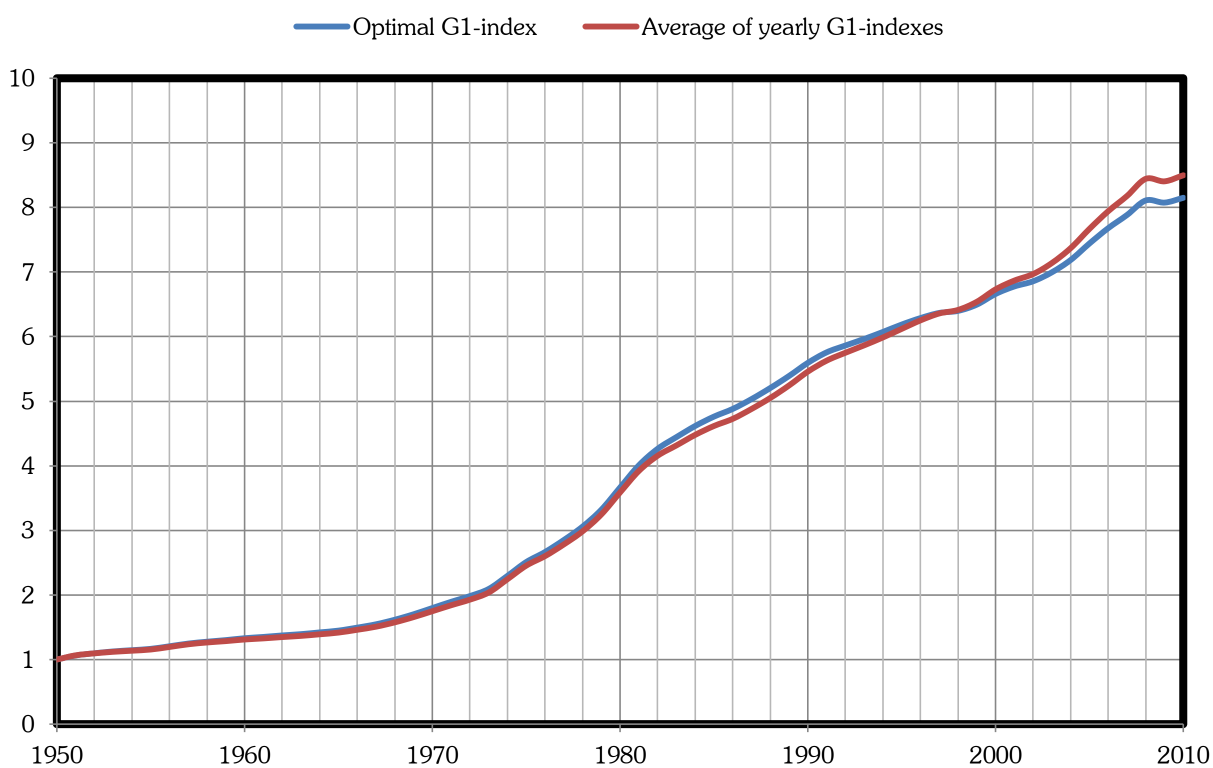


Fig. 9 Comparison of G1 indexes



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In fig.9 we show the value of the $\bar{G}_3$ index together with the $G_3$ index associated with the constant U, showing considerable agreement between the two.

## VII. Year Pairs Analysis

In our original analysis discussed in section V we have taken simultaneously into account all of the available data for the period 1950-2010. In section VI we have used part of the data in order to determine each of the locally optimal utility functions, but then we have applied each of those utility functions to the whole time interval. We will now look at the data in a different way. We select one time interval, say $t_r$ as the *reference time interval*. For each time interval $t_i$ we will determine two sets of function parameter vectors $(\mathbf{a}^{\mathbf{o}}_{\mathbf{ir}},\mathbf{c}^{\mathbf{o}}_{\mathbf{ir}})$ and $(\mathbf{a}^{\mathbf{o}}_{\mathbf{ri}},\mathbf{c}^{\mathbf{o}}_{\mathbf{ri}})$ each of which minimizes the error function

(94) $$S(i,r,\mathbf{a},\mathbf{c})=\frac{1}{b_i^2}\left[\sum_{m=1}^{M}\left[p_{im}\left[q_{im}-q_m^*\left(b_i,\mathbf{p_i}\,|\,\mathbf{a},\mathbf{c}\right)\right]\right]^2\right]+\frac{1}{b_r^2}\left[\sum_{m=1}^{M}\left[p_{rm}\left[q_{rm}-q_m^*\left(b_r,\mathbf{p_r}\,|\,\mathbf{a},\mathbf{c}\right)\right]\right]^2\right]$$

subject to the constraint, in the case of the first set, that the actual and optimal quantity vectors are exactly equal for time interval $t_r$, and for the second set that the corresponding vectors are exactly equal for time interval $t_i$. It should be noted that in the case of only a pair of time intervals it may be possible to find algebraically a pair of parameter vectors such that

(95) $$S(i,r,\mathbf{a},\mathbf{c})=0$$

However, we will use the same gradient technique used in the general case, so as to maintain the general consistency of methodology.

We can then define the *optimal time interval pair G3 Index* as

(96) $$\bar{G}_{3p}\left(i\,|\,r\right)=\sqrt{I(\mathbf{p_i},\mathbf{p_r},0.5\,|\,\mathbf{a}^{\mathbf{o}}_{\mathbf{ir}},\mathbf{c}^{\mathbf{o}}_{\mathbf{ir}})\times I(\mathbf{p_i},\mathbf{p_r},0.5\,|\,\mathbf{a}^{\mathbf{o}}_{\mathbf{ri}},\mathbf{c}^{\mathbf{o}}_{\mathbf{ri}})}$$

Note that the above can also be written as

(97) $$\bar{G}_{3p}(i\,|\,r)=\sqrt{\frac{B^*(0.5,\mathbf{p_i}\,|\,\mathbf{a}^{\mathbf{o}}_{\mathbf{ir}},\mathbf{c}^{\mathbf{o}}_{\mathbf{ir}})}{B^*(0.5,\mathbf{p_r}\,|\,\mathbf{a}^{\mathbf{o}}_{\mathbf{ir}},\mathbf{c}^{\mathbf{o}}_{\mathbf{ir}})}\times\frac{B^*(0.5,\mathbf{p_i}\,|\,\mathbf{a}^{\mathbf{o}}_{\mathbf{ri}},\mathbf{c}^{\mathbf{o}}_{\mathbf{ri}})}{B^*(0.5,\mathbf{p_r}\,|\,\mathbf{a}^{\mathbf{o}}_{\mathbf{ri}},\mathbf{c}^{\mathbf{o}}_{\mathbf{ri}})}}$$

It should be noted a formal analogy with the Fisher standard index. The key difference is that instead of using the “theoretical budget” associated with the quantities *actually purchased* at the appropriate time intervals, the index uses the *optimal* budgets associated with the predetermined value of the utility function, evaluated at the appropriate price vector. Both the Fisher standard index and the $\bar{G}_{3p}(i\,|\,r)$ defined above have the
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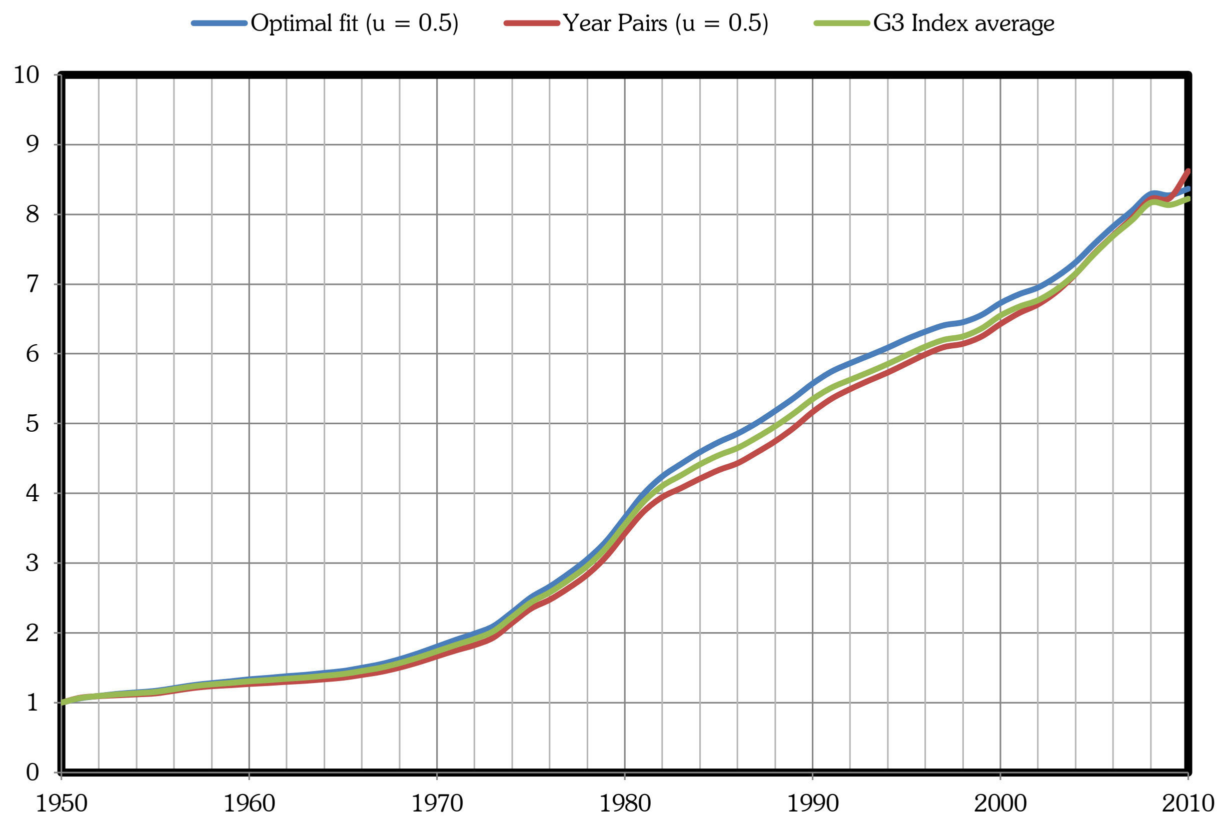
characteristic that only the data for the two specified time intervals are used, *without any reference to any intervening time intervals*.

Fig. 10



In fig.10 we show three indexes, all evaluated at the utility value of 0.5: the G_3 index defined in section IV, the $\bar{G}_3(i,1950|10)$, already shown in Fig.9 and the above defined $\bar{G}_{3p}(i|r)$. All three indexes are in very good agreement. The important fact to note here is that while the first two indexes were evaluated on the basis of the full knowledge of the whole information in the 1950-2010 period, each point of the last index was evaluated using only the information about the two time period in question. In other words, in our methodology, by using the $\bar{G}_{3p}(i|r)$ index defined above it is possible to evaluate a realistic price index for any two time intervals, without any knowledge of any intervening data, *no matter how distant the two time intervals are*. We believe that this demonstrates quite clearly the “robustness” of the approach that we have been discussing.

VIII. Conclusions

We believe that both the Fisher standard index and the Fisher chain index methodologies suffer from very disturbing anomalies and lack any theoretical underpinning. Approaches based on underlying utility functions have been discussed extensively in the theoretical

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literature, but appear to have had limited applications in actual practice. There are probably two basic reasons for the situation, namely:

- there is often a reluctance to make apparently "arbitrary" assumptions about the general form of the utility function;
- the computational complexity of "fitting" a large set of parameters may have been considered "impractical".

We believe that the first objection is not a valid one. The purpose of "models" is to allow us to derive useful criteria for determining a course of action. The "validity" of such models needs to be judged only by their usefulness (or lack thereof). There may have been some validity to the second objection before digital computers, but certainly not now.

The approach that we have presented is characterized by an underlying theoretical foundation that, once accepted, leads in a congruent way to the determination of the value of the indexes. Such indexes meet certain important requirements, namely they satisfy the transitivity, reciprocity and identity properties. Furthermore, the approach shows considerable robustness in the presence of drastic price changes. The approach allows for a variety of choices in the selection of specific indexes. This might be viewed as a drawback. We believe instead that this "embarrassment of riches" points to the fact that it is wishful thinking to believe that a "unique" characterization of the value of money may be found that is valid for any study of the issue. Furthermore, it suggests that attempts to characterize the value of money with high precision are probably misguided.

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